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Ultimate Limits to Quantum-Enhanced Optical Phase Tracking

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Abstract: We use the quantum Fisher information to show the ultimate limit to the accuracy of measurement of a fluctuating phase with a squeezed state. This measurement accuracy is achievable using an adaptive phase measurement.

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Measurement of phase is the basis of much precision measurement, with optical interferometry providing extremely accurate distance measurements [1]. In particular, gravitational wave detectors use high-power interferometers to measure displacements smaller than the width of a proton. A new development in gravitational wave detectors is the enhancement of accuracy using nonclassical states. Squeezed states are now used in the GEO600 detector [2], and squeezed states have been tested in LIGO, and are planned for Advanced LIGO. Another area where squeezed states have been demonstrated is the tracking of a widely fluctuating phase on a low-power beam [3].

The use of squeezed states or other nonclassical states enables one to beat the standard quantum limit. For measurements of a constant phase, this limit is variance scaling as $1/\bar{n}$, where $\bar{n}$ is the mean photon number. For constant phase the ultimate limit to the accuracy is the Heisenberg limit, which scales as $1/\bar{n}^2$ [4]. In the case that the phase is fluctuating, it is fairly easy to see that the corresponding standard quantum limit is $1/\sqrt{\mathscr{N}}$, where $\mathscr{N}$ is the photon flux [5]. However, the equivalent of the Heisenberg limit for a fluctuating phase is unknown; that is, what is the ultimate limit to the accuracy of measurement of a fluctuating phase, given an arbitrary state?

In this work we provide a partial answer to this question by finding the greatest accuracy that can be obtained using squeezed states. We do this by determining the quantum Fisher information, using the newly developed theory of quantum Fisher information for waveform estimation [6]. We thereby show that the best possible accuracy using squeezed states scales as $\mathscr{N}^{-2/3}$. Moreover, by using the adaptive phase measurements proposed in Ref. [7], but correcting that work to use the correct expression for the photon flux [8], we show that this accuracy is achievable.

Reference [6] shows that the variance for the estimate, $\hat{\varphi}(t)$, of a varying phase, $\varphi(t)$, can be lower bounded by

$$\langle[\hat{\varphi}(t)-\varphi(t)]^2\rangle \geq F^{-1}(t,t), \tag{1}$$

where the Fisher information, $F(t,t')$, is given by a sum of a quantum and a classical part, $F(t,t') := F^{(Q)}(t,t') + F^{(C)}(t,t')$. The quantum and classical parts of the Fisher information are given by integrals in terms of the probability distribution for the phase waveform, $P[\varphi]$, and the generator of the phase shifts, $h(t) = b^\dagger(t)b(t)$.

We consider a squeezed state where the quadratures are independent, and satisfy $\langle X(t)\rangle = 2\alpha$, $\langle Y(t)\rangle = 0$. The normally ordered correlations are given in [3], and yield

$$\begin{aligned} F^{(Q)}(t,t') &= 4\mathscr{N}\delta(t-t') + \alpha^2(R_+-1)(1-x)\gamma e^{-(1-x)\gamma|t-t'|/2} \\ &+ \frac{\gamma^2}{32}\left[(R_+-1)^2(1-x)^2 e^{-(1-x)\gamma|t-t'|} + (R_--1)^2(1+x)^2 e^{-(1+x)\gamma|t-t'|}\right], \end{aligned} \tag{2}$$

where R_- and R_+ are the squeezing and antisqueezing levels, respectively, γ is the cavity's decay rate, and for pure squeezing $x = (\sqrt{R_+}-1)/(\sqrt{R_+}+1)$.

We consider a classical variation of the phase according to a Gaussian probability distribution, in which case the classical Fisher information simplifies to $F^{(C)}(t,t') = \Sigma^{-1}(t,t')$, where $\Sigma(t,t') = \langle\varphi(t)\varphi(t')\rangle$. The inverse of the Fisher information can be found using the Fourier transform, which yields

$$\tilde{F}^{-1}(\omega) = \frac{\tilde{\Sigma}(\omega)}{1+\tilde{F}^{(Q)}(\omega)\tilde{\Sigma}(\omega)}. \tag{3}$$

Given an Ornstein-Uhlenbeck process for the phase, one obtains $\tilde{\Sigma}(\omega) = \kappa/(\lambda^2+\omega^2)$, where κ is the inverse correlation time and λ is the decay rate. It is then convenient to define rescaled quantities as $\mathcal{N}_\star = \mathcal{N}/\kappa$, $\gamma_\star = \gamma/(\kappa \mathcal{N}_\star^{5/6})$, $R_\star = R_+/\mathcal{N}_\star^{1/3}$, as well as the quantity $\tau = \gamma_\star\sqrt{R_\star}/8$, which has the allowable range $\tau \in [0,1]$. One then finds that, to leading order,

$$F^{-1}(t,t) = C\mathcal{N}_\star^{-2/3}, \tag{4}$$

where C varies as a function of τ and $\gamma_\star$. This functional variation is shown in Fig. 1, and it can be seen that the minimum is for $\tau = 1$. The minimum can be found analytically to be $C = (587-143\sqrt{13})^{1/6}/(4\sqrt{6}) \approx 0.20788$.

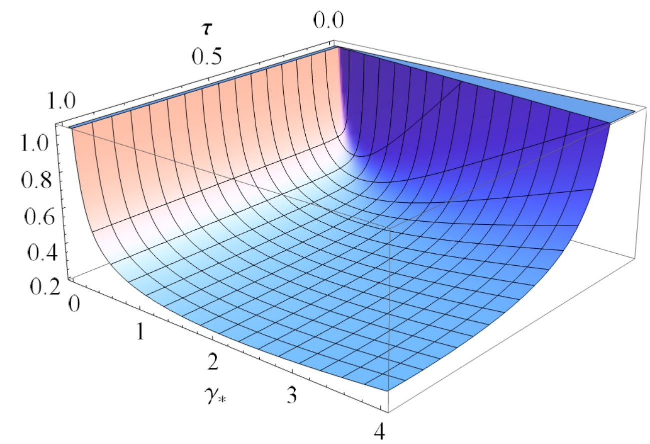

Fig. 1. 3D plot of the scaling constant for $F^{-1}(t,t)$ as a function of $\gamma_\star$ and τ.

So far we have shown that, for measurements on squeezed states, the quantum Fisher information gives a lower bound to the variance scaling as $(\kappa/\mathcal{N})^{2/3}$. This then raises the question of whether this scaling is achievable. We assume here that a large degree of squeezing and low loss can be achieved; the question is whether such measurements are achievable in principle. The difficulty is that, with extremely large squeezing, there is enormous uncertainty in the antisqueezed quadrature. This means that the measurements need to be performed very accurately on the squeezed quadrature to avoid excess fluctuations. That is, we need a scheme for very accurate real-time adaptive measurements of phase, as demonstrated in [3].

The scheme for arbitrarily high squeezing was presented in [7], but gave the scaling as $(\kappa/\mathcal{N})^{5/8}$. We have now found that the expression given for the flux due to the squeezing in [7] was not consistent with the model of squeezing, and overestimated the flux. As a result, greater squeezing can be used for a given level of flux, and the scaling for that adaptive phase measurement scheme is $(\kappa/\mathcal{N})^{2/3}$ [8], matching the lower bound. The scaling constant for that scheme is about 1, which is somewhat larger than the constant for the lower bound. Thus we have found the ultimate limit to the scaling for measurements of a fluctuating phase using a squeezed state, and shown that this limit is achievable, albeit with a slightly different scaling constant.

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