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Influence of hysteresis on groundwater wave dynamics in an unconfined aquifer with a sloping boundary

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Abstract

In this paper, the influence of hysteresis on water table dynamics in an unconfined aquifer was examined using a numerical model to solve Richards unsaturated flow equation. The model was subject to simple harmonic forcing across a sloping boundary with a seepage face boundary condition. Time series from both hysteretic and non-hysteretic models were subject to harmonic analysis to extract the amplitude and phase profiles for comparison with existing sand flume data [Cartwright, N., P. Nielsen, L. Li, 2004. Experimental observations of water table waves in an unconfined aquifer with a sloping boundary. *Advances in Water Resources.*, 27, pp. 991–1004]. The results from both model types show good agreement with the data indicating no influence of hysteresis at the oscillation period examined ($T = 348$ s). The models were then used to perform a parametric study to examine the relationship between oscillation period and hysteresis effects with periods ranging from 3 *min* to 180 *min*. At short oscillation periods, ($T \approx 180$ s) the effects of hysteresis were negligible with both models providing similar results. As the oscillation period increased, the hysteretic model showed less amplitude damping than the non-hysteretic model. For periods greater than $T = 60$ *min*, the phase lag in the non-hysteretic model is greater than for the hysteretic one. For periods less than $T = 60$ *min* this trend is reversed and the hysteretic model

produced a greater phase lag than the non-hysteretic model. These findings suggest that consideration of hysteresis dynamics in Richards' equation models has no influence on water table wave dispersion for short period forcing such as waves ($T \approx 10\text{ s}$) whereas for long period forcing such as tides ($T \approx 12.25\text{ hrs}$) or storm surges ($T \approx \text{days}$) hysteresis dynamics should be taken into account.

Keywords: Hysteresis; Groundwater waves; Unsaturated flow; Seepage face; Richards' equation

1. Introduction

A variety of coastal zone processes such as salt-water intrusion and contaminant transport into coastal aquifers (e.g. Robinson et al., 2006; Xin et al., 2010) and coastal morphology (e.g. Grant, 1946, 1948) are influenced by oceanic surface and aquifer subsurface water interaction. The boundary is subjected to a wide range of oceanic forces such as tide, wave, storm surge etc., each with varying magnitudes and frequencies. The resultant groundwater waves have been widely studied using a range of approaches. These include field observations (e.g. Lanyon et al., 1982; Nielsen, 1990; Turner et al., 1997; Raubenheimer et al., 1999), laboratory experiments (e.g. Parlange et al., 1984; Nielsen et al., 1997; Boufadel et al., 1998; Ataie-Ashtiani et al., 1999; Cartwright et al., 2003), analytical modelling (e.g. Parlange et al., 1984; Barry et al., 1996; Li et al., 2000; Kong et al., 2013); and numerical modelling (e.g. Li et al., 1997; Boufadel et al., 1998; Baird et al., 1998; Ataie-Ashtiani et al., 1999; Xin et al., 2010; Shouhstari et al., 2015).

Several modelling studies have considered the influence of moisture above the water table. In terms of theoretical development, Parlange and Brutsaert (1987) developed a capillary fringe correction for free surface flow theory based on the Green and Ampt (1911) infiltration model. This approach was further developed by Barry et al. (1996) and Li et al.

(2000). More recently Kong et al (2013; 2015) has developed approximate analytical solutions to Richards' unsaturated flow equation (Richards, 1931). Several authors have investigated periodic unsaturated groundwater flow via numerical solution of Richards' equation (e.g. Boufadel et al., 1998; Ataie-Ashtiani et al., 1999; Boufadel, 2000; Naba et al., 2002). However, none of these previous modelling efforts have considered the effects of hysteresis on groundwater wave dynamics.

The effects of hysteresis on groundwater dynamics has been examined but has been limited to studies of the dynamics of 1D vertical sand columns. Lehmann et al., (1998) demonstrated the need to consider hysteresis effects in order to model the observed temporal asymmetry in their data which was forced by a temporally symmetric saw tooth wave shape. Stauffer and Kinzelbach (2001) observed the moisture dynamics above a periodic water table and also concluded the need to consider hysteresis effects to model their observations. Werner and Lockington (2003) also demonstrated the need to include hysteresis effects when modelling the sand column data of Nielsen and Perrochet (2000a, b). More recently, Cartwright (2014) demonstrated the relationship between oscillation frequency and the degree of influence of hysteresis on moisture-pressure dynamics in a sand column.

This paper extends this previous work by examining the influence of hysteresis on a propagating groundwater wave. Here we consider a 2D vertical, unconfined aquifer system forced at one end by a simple harmonic periodic boundary condition acting across a sloping boundary. The paper is organized as follows: Section 2 provides a brief description of the experimental data of Cartwright et al. (2004) used in the model verification. Section 3 describes the numerical model including the implementation of a seepage face boundary condition. The verification of the numerical results against the existing experimental data is discussed in Section 4 and the model is then used to undertake a parametric study to examine the relationship between oscillation frequency and the degree of influence of hysteresis effects. Finally, Section 6 summarises the major findings and conclusions.

2. Experimental setup and procedures

In this paper, the experimental data of Cartwright et al. (2004) have been used for verification of the numerical model. For convenience, a brief outline of the experimental setup and procedures is given here. For full details the reader is referred to Cartwright et al. (2004).

2.1 The sand flume

The sand flume used by Cartwright et al. (2004) was 9 m long, 1.5 m high and 0.15 m wide. At one end of the flume, a simple harmonic oscillation was generated in the clear water tank connected to the sand flume. The top of the flume was exposed to the atmosphere and all other boundaries were no-flow boundaries (Figure 1).

2.2 The sand

The sand in the flume was predominantly quartz whose properties were examined in detail by Nielsen and Perrochet (2000 a, b). Cartwright et al. (2004) measured in-situ hydraulic conductivity by a slug test at different locations of the flume. Cartwright (2014) also provided the hydraulic properties for the same sand using in the sand column experiments. The summary of hydraulic properties of the sand is given in Table 1.

2.3 The driving head

The simple harmonic driving head in the clear water tank is described by,

$$H_o = d + A \cos(\omega t) \quad (1)$$

where H_o (m) is the driving head, d (m) is the mean driving head, A (m) is the driving head amplitude and

$\omega = 2\pi/T$ (rad/s) is the oscillation frequency and T (s) is the oscillation period. Table 2 summarises the driving head parameters.

2.4 The sloping boundary

An initial linear profile was set up manually and it was subjected to the driving force until a stable profile was established (~ 2 days). The slope of the profile then was determined by fitting the best straight line which had the slope as 0.205 ($\tan \beta = 0.205$ or $\beta = 11.7^\circ$). The low and high water marks (LWM and HWM) were located at $(x = 0.47 (m), z = 0.81 (m))$ and $(x = 2.45 (m), z = 1.21 (m))$, respectively.

2.5 Monitoring of piezometric head

The piezometric head in the saturated zone at different locations along the flume was measured using cylindrical piezometers extending horizontally into the sand.

3. Numerical modeling of unsaturated groundwater flow

3.1 Governing equation

Richards' equation is widely used for modelling the water movement in the unsaturated zone of porous media. The equation is a non-linear partial differential equation,

$$(C_m + S_e S) \frac{\partial h}{\partial t} + \nabla \cdot (-K_s k_r (\nabla H)) = 0 \quad , H = h + z \quad (2)$$

where $H(m) = h + z$ is the hydraulic head, $h(m)$ is the pressure head which is the dependent variable, $z (m)$ is the elevation head, $C_m (m^{-1})$ is the specific moisture capacity, $S_e (-)$ is the effective saturation, $S (m^{-1})$ is the storage coefficient, $K_s (m/s)$

is the saturated hydraulic conductivity, $k_r (-)$ is the relative permeability and $t (s)$ is the time.

The hydraulic properties (i.e. C_m, S_e, k_r) vary for unsaturated conditions (i.e. negative pressure head) and reach to a constant value at saturation condition (i.e. zero or positive pressure head). These hydraulic properties are dependent on the soil retention properties and here the analytic formulas of van Genuchten (1980) are used to quantify them.

$$\theta = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{[1 + |\alpha h|^\beta]^m} & h < 0 \\ \theta_s & h \geq 0 \end{cases} \quad (3)$$

where θ is the volumetric moisture content, θ_s and θ_r are saturated and residual moisture contents respectively.

The relative permeability is the ratio of the unsaturated hydraulic conductivity relative to the saturated value and for the van Genuchten (1980) model is given by,

$$k_r = \begin{cases} S_e^l \left[1 - \left(1 - S_e^{\frac{1}{m}} \right)^m \right]^2 & h < 0 \\ 1 & h \geq 0 \end{cases} \quad (4)$$

where the effective saturation is,

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (5)$$

The specific moisture capacity is defined as,

$$C_m = \frac{d\theta}{dh} = \begin{cases} \frac{\alpha m}{1 - m} (\theta_s - \theta_r) S_e^{\frac{1}{m}} \left(1 - S_e^{\frac{1}{m}} \right)^m & h < 0 \\ 0 & h \geq 0 \end{cases} \quad (6)$$

where α (m^{-1}), β (-), $l = 0.5$ (-) and $m = 1 - 1/\beta$ are empirical curve fitting parameters and $h = 0$ corresponds to the water table position .

Table 1 provides the hydraulic and van Genuchten moisture retention parameters used in the numerical simulation.

In this paper, the Richards equation has been solved by the finite element method using the FEFLOW 6.0 software package (FEFLOW, 2012).

3.2 Seepage face boundary condition

Generally, in the falling tide stage, the speed of movement of the water table is less than the ocean level; hence, the water table exit point becomes decoupled from the ocean level and a seepage face is formed and has a glassy appearance. When decoupling occurs, two distinct pressure zones exist: (1) along the seepage face where the water table is at the sand surface and the pressure equal atmospheric pressure ($h = 0$) ; (2) above the exit point where the sand surface has a matt appearance the pressure is negative due to the formation of meniscuses (Figure 2).

To consider the formation of seepage face at the beach face, a mixed boundary condition was applied using a prescribed head with flux constraint method in the FEFLOW simulations. The prescribed head with flux constraint method is similar to Clement et al.'s (1994) method and switches the boundary

condition between Dirichlet and Neumann based on the flow direction on each part. For boundary nodes below the low tide level, the head is the same as the driving head level (H_o). For boundary nodes above the high tide level a no-flow condition is applied (Figure 2).

For boundary nodes in the inter-tidal zone, the prescribed head with flux constraint is applied. In each time step, if the shoreline position is above the node then the head is the same as the driving head and the flux is unconstrained. If the shoreline position is below the node then the node will either be in the seepage face (outflow from the domain) or above the exit point (no-flow). If the flow at the node is positive (i.e. into the domain) then the prescribed head condition is relaxed and the pressure head is allowed to be negative. The model then iterates and adjusts the water table position until the solution converges. For further details of this method the reader is referred to Shoushtari et al., (2015).

3.3 Hysteresis effects

There are different models to describe hysteresis in the retention curves (e.g. Scott et al., 1983; Kool and Parker, 1987; Lenhard and Parker, 1987; Šimůnek et al., 2009) which can be categorised in two groups, physically based models and empirical models. Among them, empirical models have more robustness and flexibility. Here, the empirical model of

Scott et al. (1983) is used where the primary, secondary and higher-order scanning loops can be scaled from the main drying and wetting moisture retention curves. In this method, drying scanning curves are obtained by using the van Genuchten parameters vector $(\theta_s^*, \theta_r, \alpha_d, \beta)$ in equation (3), where θ_s^* replaces θ_s , α_d is the van Genuchten parameter for the first drying curve and θ_s^* denotes the saturated moisture content obtained from passing the main drying curve through the reversal point

$$\theta_s^* = (\theta_\Delta - \theta_r)[1 + |\alpha_d h_\Delta|^\beta]^m + \theta_r \quad (7)$$

where θ_Δ is the moisture content at the reversal point on the main drying curve at the reversal pressure h_Δ .

In a similar manner any wetting scanning curve can be obtained by using the van Genuchten parameters vector $(\theta_s, \theta_r^*, \alpha_w, \beta)$ in equation (3), where α_w is the adopted van Genuchten parameter for the first wetting curve and θ_r^* denotes the residual moisture content obtaining from passing the main wetting curve through the reversal point

$$\theta_r^* = \frac{\theta_s - \theta_\Delta [1 + |\alpha_w h_\Delta|^\beta]^m}{1 - [1 + |\alpha_w h_\Delta|^\beta]^m} \quad (8)$$

where θ_Δ is the moisture content at the reversal point on the main wetting curve at the reversal pressure h_Δ . In this model all scanning loops have the form of equation (3). For further details refer to Scott et al. (1983) or to Diersch (2014).

4. Model-data comparison

To compare the numerical results with the experimental data, the oscillation amplitudes (R) and phases (ϕ) for the first three harmonics were extracted using harmonic analysis. Consistent with the data of Cartwright et al. (2004), the third harmonic was negligible ($R_3/A < 3\%$) and has therefore been excluded from any further analysis.

4.1 Amplitude profile

Figure 3 shows the comparison between simulated and measured amplitudes extracted for the first two harmonics. The first model was non-hysteretic with the experimentally determined van Genuchten parameters given in Table 1 ($\alpha = 2.3 \text{ (m}^{-1}\text{)}$, $\beta = 10$, $\theta_s = 0.38$ and $\theta_r = 0.08$). As shown in Figure 3, for $x < 4.5 \text{ m}$, the model accurately predicts amplitudes for the first two harmonics. The model also captures the generation of the 2nd harmonic in the inter-tidal zone, accurately predicting the maximum amplitude and its location. For $x > 4.5 \text{ m}$, the model overpredicts the amplitude decay rate but, noting the semi-logarithmic scale, this difference is no more than 1 cm and for $x > 7 \text{ m}$, the observed amplitude is less than 1 cm which is approaching the accuracy of the measurements.

The dotted lines in Figure 3 represent the model results including hysteresis with $\alpha_d = 2.3 \text{ (m}^{-1}\text{)}$; $\beta = 10$ and

$\alpha_w = 3.91 \text{ (m}^{-1}\text{)}$ based on the hysteresis ratio $\zeta = \alpha_w/\alpha_d = 1.7$ recommended by Kool and Parker (1987). It is clear that very little difference between the hysteretic and non-hysteretic model results exists. This finding is consistent with the sand column experiments of Cartwright (2014) which demonstrated that, for shorter oscillation periods ($T < 30 \text{ min}$), the observed moisture-pressure dynamics were non-hysteretic. It was noted that the average slope of the observed moisture-pressure curve (i.e. the average capillary capacity), was closely matched with a non-hysteretic moisture-pressure curve with $\beta = 3$. To further investigate this, a non-hysteretic model with $\alpha = 3 \text{ (m}^{-1}\text{)}$ and $\beta = 3$ was applied. The results are also shown in Figure 3 (bold and thin dashed lines for the first and second harmonics respectively) where the model-data comparison in the inter-tidal zone is very similar to the other two models whilst the performance landward of $x = 4.5 \text{ m}$ is worse.

Table 3 provides a summary of RMS errors for each simulation. As it can be seen in this table, the RMS errors for all three models are very small i.e. $(0)10^{-2} \text{ m}$ and it varies from 1.5 to 10 % of the driving head amplitude (A).

4.2 Phase profiles

Figure 4 shows the model-data comparison for all three models which all perform reasonably well for $x \leq 4 \text{ m}$. For $x > 4 \text{ m}$ there is some divergence between the models. For the first

harmonic (bold lines and circles), the hysteretic model is seen to perform better than the non-hysteretic $\beta = 10$ model but interestingly the non-hysteretic $\beta = 3$ model performs the best. For the second harmonic (thin lines and squares), the non-hysteretic $\beta = 3$ model again performs the best and the hysteretic and non-hysteretic $\beta = 10$ models are essentially the same. The RMS errors for each model are summarised in Table 3.

4.3 Water table exit point

Using the seepage boundary condition in the model allows decoupling between the driving head and the water table. Figure 5 compares the observed and predicted exit point elevation time series for hysteretic ($\alpha_d = 2.3 \text{ m}^{-1}$, $\alpha_w = 3.91 \text{ m}^{-1}$ and $\beta = 10$) and non-hysteretic ($\alpha = 3 \text{ m}^{-1}$ and $\beta = 3$) models. The results for the non-hysteretic model with $\alpha = 2.3 \text{ (m}^{-1}\text{)}$ and $\beta = 10$ was virtually identical to the hysteresis model so it has not been shown here. The non-hysteretic model $\beta = 3$ can predict the exit point location better than the two $\beta = 10$ models however there is a degree of uncertainty with observed exit point which was inferred from sub-surface pressure measurements by Cartwright et al. (2004).

5. Parametric analysis

In section 4, it was found that hysteresis had no discernible influence on the groundwater dynamics for the oscillation

period corresponding to the existing experimental data ($T = 348$ s). To further investigate any potential influence of hysteresis on groundwater wave behaviour a parametric study using the numerical models is presented here. In particular, the relationship between the influence of hysteresis and oscillation period is examined. The hysteretic ($\alpha_d = 2.3$ (m^{-1}), $\alpha_w = 3.91$ (m^{-1}) and $\beta = 10$) and non-hysteretic ($\alpha = 2.3$ (m^{-1}) and $\beta = 10$) models were applied for 12 different oscillation periods ranging from $T = 3$ min to $T = 180$ min with all other parameters kept the same. It should be noted that, since the water table wave length increases with increasing period, the length of numerical domain has been expanded from 9 m to 30 m to avoid any influence of reflection from the no-flow boundary condition at the landward end of the domain.

5.1 Amplitudes and phases

The amplitudes and phases from each simulation have been extracted using harmonic analysis and the results are shown in Figures 6 and 7. For the sake of clarity, in these figures only the results for the first harmonics at $z = 0.8$ m for three selected periods (i.e. $T = 348$ s, 60 min, 180 min) are presented.

Figure 6 shows that all models are very similar in the inter-tidal zone suggesting negligible influence of oscillation period or hysteresis in this region. Landward of the high water mark the different models begin to diverge. As discussed in detail in

section 4, for the short period $T = 348 \text{ s}$ (solid lines), the amplitude profile extracted from hysteretic and non-hysteretic model is similar to each other which indicates that hysteresis does not have a significant influence for short oscillation periods. When the period is increased, the hysteretic and non-hysteretic model results begin to diverge as the water table wave travels further into the aquifer. In these cases, the hysteretic model shows less damping than the non-hysteretic model. For $T = 60 \text{ min}$ (dashed lines), the amplitude drops below 2 mm at $x \approx 6.9 \text{ m}$ for the non-hysteretic model and at $x \approx 10.5 \text{ m}$ for the hysteretic model. For $T = 180 \text{ min}$ these two distances increase to $x \approx 9.5 \text{ m}$ and $x \approx 16.5 \text{ m}$, respectively.

Figure 7 shows the comparison of the simulated phase profiles. For $x > 3 \text{ m}$, all model results are virtually identical further supporting the lack of influence of oscillation period or hysteresis in and near the inter-tidal zone. For $x > 3.5 \text{ m}$, the phase profiles for the hysteretic models (bold lines) starts diverging from the non-hysteretic models (thin lines). For $T = 348 \text{ s}$, the hysteretic model (solid bold line) shows bigger phase lag than non-hysteretic model (thin solid line), while this trend reverses for $T = 180 \text{ min}$ i.e. hysteretic model (bold dotted line) obtains smaller time lag than non-hysteretic model (thin dotted line). For $T = 60 \text{ min}$ both models show almost the same trend

for the phase lag (bold and thin dashed line for hysteretic and non-hysteretic model, respectively).

5.2 Water table wave number

To analyse the divergent behaviour of the model results landward of the inter-tidal zone, the following section discusses the behaviour of the water table wave number k which describes the dispersion of a groundwater wave as follows. The general form of the water table wave in response to simple harmonic forcing is (e.g. Nielsen, 1990),

$$\eta(x, t) = Ae^{-k_r x} \cos(\omega t - k_i x) \quad (9)$$

where η is the water table elevation and the water table wave number $k = k_r + ik_i$. k_r describes the amplitude decay rate and k_i describes the rate of increase in phase lag with distance travelled landward and i is the imaginary number.

Separation of the amplitude and phase terms in equation (9) and rearranging yields (Nielsen, 1990),

$$k_r x = -\ln \left[\frac{|\eta(x)|}{A} \right] \quad (10)$$

$$k_i x = \phi(x = 0) - \phi(x) \quad (11)$$

Therefore, by plotting the right hand sides of equations (10) and (11) as a function of x , the slope of the lines of best fit will yield the real and imaginary parts of the wave number respectively. This approach was applied to each of the model results and the results are shown in Figures 8 and 9 where the

wave numbers are plotted as a function of the non-dimensional aquifer depth ($n\omega d/K$).

Figure 8 shows that the amplitude decay rates for the non-hysteretic simulations are generally greater than the hysteretic model. A significant difference between hysteretic and non-hysteretic models exist for small non-dimensional aquifer depths (i.e. long periods) but the non-dimensional aquifer depth increases (i.e., decreasing period), the difference between hysteretic and non-hysteretic becomes smaller and the difference is almost zero for non-dimensional aquifer depth of 52 (i.e. $T = 348$ s).

Figure 9 shows two different behaviours for k_i . For non-dimensional aquifer less than ~ 5 (i.e. $T \approx 60$ min) the non-hysteretic phase lag is greater than the hysteretic one but, by increasing the non-dimensional aquifer depth this trend is reversed and the hysteretic model obtains greater phase lag. Based on the decreasing trend of the hysteretic model for larger non-dimensional aquifer depth, it seems that both models provide the same amount of phase lag or k_i for large non-dimensional aquifer depth.

These results show that considering hysteresis in water retention does not have significant effect on water table wave propagation for shorter periods (e.g. wind waves, $T \approx 10$ s) and the same results can be obtained using a non-hysteretic model. For long periods however (e.g. tides, $T \approx 12.25$ hrs or storm

surges, $T \approx \text{days}$), hysteresis does influence the dispersion of water table waves. At these periods, it is therefore likely that hysteresis effects will influence models of contaminant transport and saltwater intrusion in unconfined aquifers.

5.3 Scanning loops

Figure 10 shows a sample of moisture-pressure loops for $T = 60$ and 180 min in different locations along the sand flume midway between the high water mark (HWM) and the sand surface i.e. $z = 1.35 \text{ m}$. It is clear that the size of the scanning loop is increased by increasing the period which is in agreement with Cartwright's (2014) sand column observations. Also, the extent of hysteresis in the moisture-pressure loops is greatest near the forcing boundary and the loops rapidly collapse and the moisture-pressure curves become non-hysteretic. For $T = 60 \text{ min}$ at $x \approx 5.3 \text{ m}$ the loop almost disappears and moisture-pressure dynamics become non-hysteretic and for $T = 180 \text{ min}$ this characteristic occurs at $x \approx 8.3 \text{ m}$ due to the greater penetration of the longer period wave. This figure confirms that hysteresis effects become important for large periods and it should be taken into account in water table dynamics.

Also shown in Figure 10 is a representative non-hysteretic $\beta = 3$ curve which has a very close comparison with the average slope (i.e. the average capillary capacity) of the

scanning loops. This is consistent with the sand column observations of Cartwright (2014).

6. Conclusion

The influence of hysteresis in moisture-pressure dynamics on the dispersive properties of groundwater waves has been investigated using a numerical solution of Richards' unsaturated groundwater flow equation with a seepage face boundary condition. The model was forced with a simple harmonic boundary condition acting across a sloping boundary. The model was verified against the sand flume data of Cartwright et al. (2004) by comparing the modelled and observed amplitude and phase lag profiles for the first two harmonics. For the experimental period of $T = 348$ s, there was no significant difference between hysteretic and non-hysteretic models. The RMS errors for all simulations were less than 10 % of the driving head amplitude (A) and the driving force period (T) for amplitude and phase profiles respectively. Hence, it can be concluded that, for this period ($T = 348$ s), either hysteretic or non-hysteretic models can adequately predict groundwater wave propagation.

To further study the effects of hysteresis on water table wave propagation, a parametric analyses was undertaken for oscillation periods ranging from $T = 3$ min to $T = 180$ min. The results demonstrate that, as the period increases, the

hysteretic and non-hysteretic model results diverge with the hysteretic model showing less amplitude damping than the non-hysteretic model. In terms of the phase lag, for the shorter period ($T = 348 \text{ s}$) the hysteretic model shows bigger phase lag than the non-hysteretic model, while this trend reverses for the longer periods (i.e. $T = 180 \text{ min}$) with the hysteretic model having a smaller time lag than the non-hysteretic model.

The results of the parametric study indicate that the influence of hysteresis on groundwater wave dispersion is not significant for short period such as wind waves ($T \approx 10 \text{ s}$) and that similar results can be obtained using a non-hysteretic model. However, for long period oscillations such as tides ($T \approx 12.25 \text{ hrs}$) and storm surges ($T \approx \text{days}$) the influence of hysteresis becomes important and should be implemented.

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