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Impact of place value chart app on students' understanding of bundling and unbundling

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Place value understanding is recognised as a critical component in the development of number understanding for young children. In this paper we investigate the usefulness of a purpose built app in supporting children's understanding in German and Australian contexts. In particular, we investigate whether the app supports the development of bundling and unbundling. Our findings indicate positive and negative aspects of the use, which have implications for both teachers and app designers.

Keywords: apps, place value, curriculum, professional development

In addition to the development of an ordinal and a cardinal concept of numbers, and the part-whole concept, the concept of place value is an important component of children's early learning in mathematics. Our number system is based on five principles: the principle of bundling and unbundling; the decimal system; the principle of place value; the multiplicative principle; and the additive principle (Kortenkamp & Ladel, 2014; Ross, 1989). Teachers require a complete understanding of the didactical concept of place value, as well as compatible working materials (e.g. place value apps) to use, to assist children in understanding place value. Whilst there are other apps that are designed to teach place value (Number Pieces; PV MAB), here we focus only on the Place Value Chart app as it was the one used in the research.

The importance of place value

The decimal number system is a powerful tool for writing mathematics and doing arithmetic as any rational number can be written using only ten different digits in a unique way (Larkin et al., 2019). The starting point of place value is the notion of unit – in our system ten ones form a new unit. This underlying process of repeated bundling means that we can represent whole numbers greater than nine by bundling in tens, and tens of tens, and tens of tens of tens… until no further bundling is possible (Houdement & Tempier, 2019; Kortenkamp & Ladel, 2014). Researching the teaching of place value is necessary. Fuson (1990, p. 345) reports that "less than 50% of third graders in the National Assessment of Educational Progress (NAEP) could do items identifying the hundreds digit, and only 65% identified the tens digit correctly". Rogers (2012, p. 648) notes that "despite the unchanging and recursive nature of our base-ten system, it seems some students never manage to fully unravel the hidden code that underlies place value". Kortenkamp and Ladel (2014, p.35) indicate the prevalence and persistence of misconceptions identifying that place value "is difficult to understand and to teach".

Place value in the German and Australian curriculums

The nationwide German curriculum is substantiated in special curriculums of the separate federal states. As an example, in the curriculum of Baden-Württemberg place value is mentioned at several junctures. One instance indicates that students should be able to use the decimal place value system and to recognise its structure (ones, tens, hundreds, bundling, unbundling). In grade 1/2 the number range is up to 100. The number range in grade 3/4 is up to 1000. For grade 3/4 the aim in the sub-strand "numbers and operations" is that the student should be able to use the building of the decimal place value system and to recognise and understand its structure (ones, tens, hundreds – as group of three, thousands, ten thousands, hundred thousands, million; bundling, unbundling) (Ministerium für Kultus, Jugend und Sport, 2016, p. 24). Similar content is incorporated in the curriculum documents of Brandenburg (LISUM, 2015). Likewise, the Australian curriculum: Mathematics (ACARA, 2018) is prescriptive in relation to place value, indicating the range of numbers that children work with. In year 1 children work with numbers up to 100; year 2 up to 1 000; year 3 up to 10 000; year 4 up to 100 000. In effect, one "place" is added each year.

Place value chart app

The app "Place Value Chart" (Kortenkamp, 2012-2018) shows a virtual place value chart, with tokens that can be moved between columns, to represent different amounts. In the app, numbers are represented by touching the screen and thus creating tokens. An example: to represent the number five, the user has to tap five times (or simultaneously once with five fingers), in the corresponding column, in this case the Ones-column. Tokens can be deleted by moving them out of the chart or by shaking the iPad. The app focusses on bundling and unbundling: by moving one token to the adjacent column to the right, the token will be unbundled into ten tokens. Moving tokens to the adjacent left column bundles ten tokens into one token in the adjacent column (N.B. only if more than ten tokens of the smaller value are available). In this way moving tokens in the app creates a change in representation, e.g. 234 can be represented as 2H 3T 4O or as 1H 12T 14O. The app offers various settings including as language, number of columns, column labels, counting base, and word/symbolic representations of totals.

Limitations of current curriculum approaches to PV and research questions

Both German and Australian curricula suggest that students learn about place value in a rigid way – with prescribed upper limits to the numbers with which children should work at each year level. We argue that this approach has detrimental impact on how children develop generalised understanding of place value, rather than an understanding determined by (and in our view limited by) the number of places prescribed in the curricula. This likely results in piecemeal, context specific knowledge (10 ones = 1 ten; 10 tens = 1 hundred) rather than generalisable knowledge (10 units always equals one unit in the adjacent column to the left). Two questions guided this research: 1. *Were children in year 1 and 2 capable of generating new columns for* *standard partitioning?* and 2. *In what ways did the app support children in understanding bundling and unbundling?*

METHODOLOGY

School 1 is a rural school in Baden-Württemberg, Germany. 43 students from two different classes at the end of year 2 took part. One classroom teacher taught both classes. School 2 is a rural school in Brandenburg, Germany. 61 students from three different classes at the start of year 3 took part and each class was taught by their own teacher. School 3 is an urban school in Queensland, Australia. 130 students in eight classes took part, four from year 1 (59) and four from year 2 (71). Each class was taught by their own teacher. The schools selected were a convenience sample located near each respective university. Teachers at the schools self-selected to be part of the research. The teaching consisted of three lessons of approximately 45-60 minutes each. The content of the lessons included bundling and unbundling, standard and non-standard representations of numbers, the need to add a column (hundreds) when representing three digit numbers in standard representation, and using the app.

Research design

The researchers met with teachers and school leaders at each of the schools. The researchers and teachers collaboratively planned a series of place value lessons that incorporated the use of the app. At this stage teachers also received a one-hour professional development session on place value, delivered by a member of the research team. All students with permission to participate completed a post intervention test (this test replicated the activities that students had been completing on the app e.g. they were required to represent various numbers on paper, using physical circular stamps that “mimicked" the use of tokens on the app). These tests were then marked by the researchers (correct response, incorrect response, no response). A range of codes were assigned to the incorrect responses indicating a typology of errors. RQ1 addresses the issue of whether or not children added an additional column when required for standard representation. RQ2 addresses the impact of the app on student performance in the post intervention test and is answered from test data and device data logs from the school in Brandenburg.

Data collected

Data collected include: Four sets of post-intervention tests that assessed whether children added an additional column when required (e.g. 97 + 5 where only the tens and ones columns are shown; Two sets of video data children completing the lessons; and one set of matched iPad data to posttests (Brandenburg school). The different data collected - i.e. video or data logs - is a consequence of differences in ethical clearance from the different schools.

The findings

The project generated a number of very interesting findings in relation to the children’s understanding of place value. However, we focus here only on whether the affordances of the app assisted (or limited) children's understanding of the general structure of place value (bundle and create a new column) and standard partitioning (i.e. use the least number of tokens to represent an amount). In this article we focus on the results of the component of the post-intervention test that required children to add an additional column to represent a number in its standard representation. The items 2a, 2b, 2c and 2d were structurally similar: In an initial place value chart with two (T,O) or three (H,T,O) columns a number x was represented by tokens. Then the children were asked to represent the number $x+y$ for a given y in a second, empty chart with the same number of columns *with the least possible number of tokens*. The students answered the questions by using a stamp that created tokens.

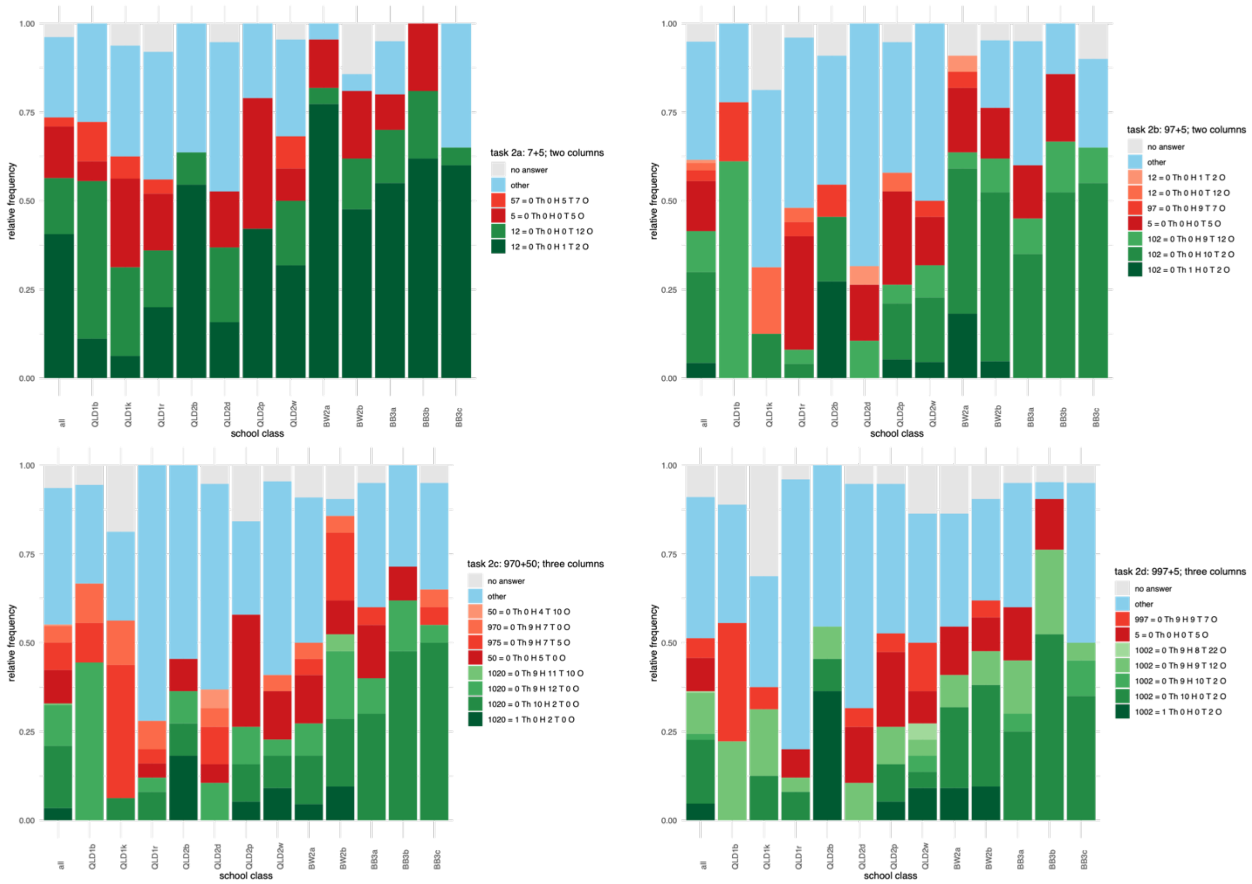

Figure 1: Students' answers to task 2a-d

Tasks 2b–d were constructed in a way that it was only possible to create a standard representation (and thus the representation with the least number of tokens overall) by adding a column (space was provided for children to do so). In Figure 1 we show the students' results grouped by school (QLD1 = Queensland school, end of year 1; QLD2 = Queensland school, end of year 2; BW = Baden-Württemberg school, end of year 2; BB = Brandenburg school, start of year 3) and class id. The bars show the frequency of answers in relation to the number of children in a class. We only distinguish answers that result in a represented number that was stamped by more than 2.5% (at least 6 children) of the overall number of children ($n = 234$). The green bottom bars depict the number of correct results, where an answer is "correct" if the number represented matches the result of the addition (e.g. 7+5 = 1T2O or 12O). The "best" answer is the one with the least number of tokens, which could only be obtained in tasks 2b–d by adding a column, and is the bottommost bar in dark green.

The most apparent and relevant results were: Adding a column was done rarely, apart from class QLD2b (No student at BB did it, but many created a maximum bundling with the available columns); many wrong answers were created across almost all classes by stamping the second summand only (dark red bars); and there was a great deal of variety amongst the classes in relation to their bundling.

In a second step, we connected the log data from the iPad to individual children and their test results for students in Brandenburg. In this way, we can analyse how individual children worked with the app and connect it to their test results. In order to better understand the cross-task behaviour, we grouped the correct answers by partition type: *Standard* is an answer where at most 9 tokens are used for each place, i.e., the standard representation of a number; *Max bundling* is an answer where at most 9 tokens are used for each place except the highest one, i.e. the representation closest to a standard representation of a number if there are not enough columns (e.g. 102 is 10T2O); *Incomplete bundling* is an answer where there are more than 9 tokens in any place except the highest one, i.e. the representation could be transformed into a representation that uses less tokens without adding a column (e.g. 102 is 9T12O). As no student in Brandenburg added any columns, the only task where they could achieve a standard partition was task 2a. The second-best answer they could give in 2b-2d was a max bundling. Incomplete bundling, on the other hand, is an indication of a missing component of place value understanding (rather than potentially a consequence of being unsure as to whether adding another column is permissible).

The log files of the app contain detailed and time-stamped information about all user interaction and configuration data of the app. In particular, they contain information about student's actions for: changing the number by *adding* or *removing* tokens (CREATE, REMOVE); *rearranging* tokens in a column without changing the value (REARRANGE); *successful* and *unsuccessful* bundling by moving a token to a column on the left (BUNDLING, REJECTED); *unbundling* by moving a token to a column on the right (UNBUNDLING); and *clearing all tokens* by shaking the iPad (SHAKE).

While the detailed information can give deep insights into what students actually did, we wanted to quantitatively measure the degree of meaningful interaction. We achieved this by counting the frequency of certain actions. Children who worked more with the app had the potential to achieve more interactions, so this measure is dependent on the teaching and thus the classroom under consideration. However, as we are investigating a possible connection of overall interactions in the app to the achievements in the test, it is feasible to combine all three groups.

We compare the distribution of interactivity ratings (that is, the frequency count of actions over all three lessons for a student) with the partition types in correct answers of these students. In the case of the SHAKE action, which is the least intentional action, we expect to see no difference in the distributions for the three types. Figure 2 shows the boxplots of the distributions and confirms this ad-hoc hypothesis.

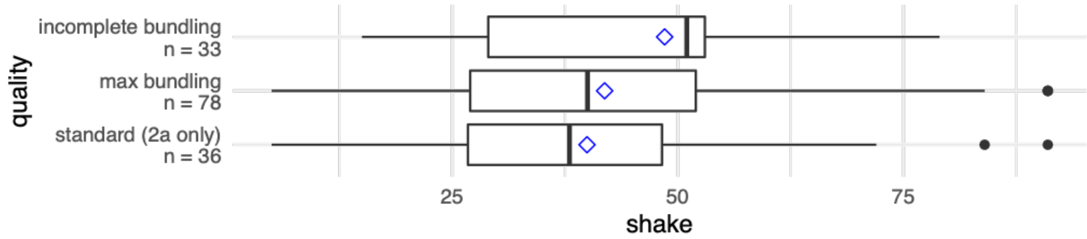

Figure 2: Distribution of SHAKE actions

For the SHAKE action, we see similar distributions of its frequency in all three conditions. On the other hand, the other actions listed above showed a different picture, similar to Figure 3, where we can see that the activity frequency distribution of students who bundled incompletely is lower compared the ones of those who found *standard* partitions or *max bundlings*.

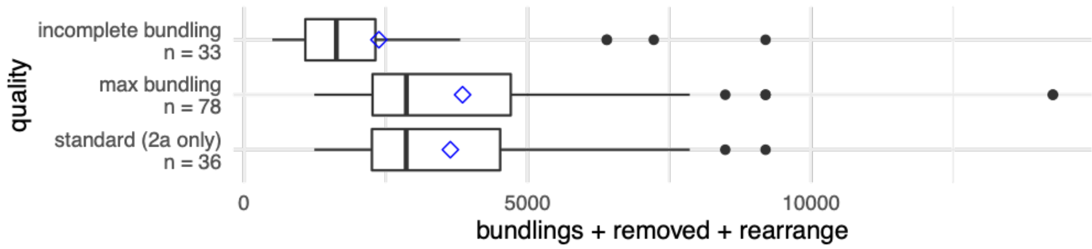

Figure 3: Distribution of intentional actions

The interaction measure we used in Figure 3 is the sum of the frequencies of (successful) bundling, removal of single tokens, and rearranging within a column. We chose these three actions as they are most likely to be intentional. Of the actions that are logged, these appear to be the most intentional ones to us. The lower quartile $Q_1 = 2267.75$ of the *max bundling* group and the lower quartile $Q_1 = 2256.75$ of the *standard* group are close to the *upper* quartile $Q_3 = 2318$ of the *incomplete bundling* group, showing that the interaction of those who performed better on the test is usually higher than those who did not perform as well.

DISCUSSION

Classifying the answers, we see a variety of representations of correct (numerical) and incorrect results. Most of the incorrect results occurred only a few times, often just once. This is unsurprising as we included tasks that are beyond the expected place value students' competencies. We can, however, still identify some typical mistakes. The most striking one is stamping only the summand (5 or 50) instead of the sum. This can be caused by misreading or misunderstanding the task. In most classes, about 10-15 % of the students made this mistake. However, two of the classes did stand out: In QLD2b the mistake occurred only with task 2c, where the two-digit number 50 had to be added, and in BB3c this mistake did not occur at all. The reason for this can be either that the students had less trouble reading and understanding the task text, perhaps caused by a classroom culture that pays more attention to reading exactly, or they might have been given additional instructions by the teacher when the test was administered. As we have no video data from the test sessions, we cannot identify the true reason.

In the first task, 2a, most students successfully did a *max bundling* if they obtained correct numerical results. With the year 1 classes, many children answered 57: This shows that they did not have a place value understanding yet, but stamped both summands individually in separate columns. This is an expected problem if place value understanding is still developing. From the data we see that the German classes performed better than the average on all tasks. In Australia, both the year 1 class QLD1b and the year 2 class QLD2b performed at or better than the average, while the other classes performed below average on all tasks. So, while we would expect German classes perform better due to their longer school experience, we also observe that younger children, even year 1 students, can show the same or better performance.

Our initial question investigated whether children would add an additional column to represent a total with the least number of tokens (often requiring the addition of a column). We do not see this behaviour in general. Out of the 234 students, only 17 added a column in one of the tasks; 10 of them did this in all tasks where necessary, 5 did it just once, and 2 did it in two out of three tasks. It is not possible to trace this to either teaching or the use of the Place Value app as, although no students in Brandenburg added a column, they performed better than other groups if we just look at the numerical result without taking the representation into account.

In addition to the possibility that some children were unsure whether they were 'allowed' to add a column in the paper test version, we also see that the app itself imposes a constraint on students' actions. As the number of columns can only be changed by accessing system preferences, separate from the app, the action of adding a column is not within easy reach. It is not a common action to "just add a column", even though the students know that it can be done. This result might imply that the app is helpful for developing a proper place value understanding, as this should encourage students to do *max bundling* of a number until they arrive at the *standard* representation. If there are not enough columns, then the idea of repeated bundling would cause adding columns to the left until every column is holding less than 10 tokens. On the other hand, inspecting the data more carefully we see that many students did a *max bundling*, i.e., they tried to be as close to a standard representation as they could without adding a column. And, when asked, the students knew that another column to the left will enable them to continue the bundling process.

The data collected from iPad logs, only available for BB3a-c, supports the theory that students who did more intentional actions within the app exhibited a better place value understanding in the test. Comparing the distributions of activity for students who did a *max bundling* in all tasks 2b–d with the distributions of activity for students who found the *standard* representation in 2a or those who did not bundle completely, we see that the distributions matches only in the first case. This means, that the activity of students who tried to bundle as much as possible matches the activity of those who found the *standard* representation in task 2a, while the group of lower-performing students (in terms of bundling) showed lower activity.

CONCLUSIONS AND IMPLICATIONS

Our findings show that merely using the Place Value Chart app in the classroom is not a guarantee for the development of place value understanding. Some of the findings might be influenced by side effects of test administration. In future research it will be necessary to standardise how the test is administered to the students. We also found that there could be a design issue of the app, in combination with a pedagogical issue: Students did not add columns to the charts in the test, although from video data we saw that they knew that it is necessary. On the positive side, the group of students who did *max bundlings*, was the group that had a higher frequency of targeted activity in the app than those who failed to bundle.

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