

Distant entanglement protected through artificially increased local temperature

Author

Carvalho, Andre R. R., Santos, Marcelo F.

Published

2011

Journal Title

New Journal of Physics

Version

Version of Record (VoR)

DOI

[10.1088/1367-2630/13/1/013010](https://doi.org/10.1088/1367-2630/13/1/013010)

Rights statement

© The Author(s) 2011. This is an Open Access article distributed under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 (CC BY-NC-SA 3.0) License (<http://creativecommons.org/licenses/by-nc-sa/3.0/>) which permits unrestricted, non-commercial use, distribution and reproduction in any medium, providing that the work is properly cited. If you alter, transform, or build upon this work, you may distribute the resulting work only under a licence identical to this one.

Downloaded from

<http://hdl.handle.net/10072/342745>

Griffith Research Online

<https://research-repository.griffith.edu.au>

Distant entanglement protected through artificially increased local temperature

Andre R R Carvalho^{1,4} and Marcelo França Santos^{2,3,4}

¹ Australian Centre for Quantum-Atom Optics, Department of Quantum Science, Research School of Physics and Engineering, The Australian National University, ACT 0200, Australia

² Departamento de Física, Universidade Federal de Minas Gerais, Belo Horizonte, Caixa Postal 702, 30123-970, MG, Brazil

³ Centre for Quantum Technologies, National University of Singapore, 2 Science Drive 3, 117542 Singapore, Singapore

E-mail: andre.carvalho@anu.edu.au and msantos@fisica.ufmg.br

New Journal of Physics **13** (2011) 013010 (11pp)

Received 19 October 2010

Published 11 January 2011

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/13/1/013010

Abstract. In composed quantum systems, the presence of local dissipative channels causes loss of coherence and entanglement at a rate that grows with the temperature of the reservoirs. However, here we show that if temperature is artificially added to the system, entanglement decay can be significantly slowed down or even suppressed conditioned on suitable local monitoring of the reservoirs. We propose a scheme for implementing joint reservoir monitoring applicable in different experimental setups, such as trapped ions, circuit and cavity QED or quantum dots coupled to nanowires, and we analyze its general robustness against detection inefficiencies and the non-zero temperature of the natural reservoir.

⁴ Authors to whom any correspondence should be addressed.

Contents

1. Introduction	2
2. Preserving entanglement through local reservoir measurements	3
2.1. Dynamics in terms of quantum jumps	3
2.2. Dynamics in terms of diffusive unravellings	5
3. Engineering and monitoring the environment	6
4. Analysis of imperfections	8
Acknowledgments	10
References	10

1. Introduction

Decoherence is considered the villain in quantum information processing, usually dragging previously entangled systems into separable ones. Over the last few decades, different approaches to overcoming its effects have been explored, including quantum error correction codes [1]–[4], dynamical decoupling [5] and encoding the system in decoherence free subspaces [6, 7].

Some other strategies make use of the decoherence process itself to try to protect entangled states. Reservoir engineering techniques, for example, explore the production of tailor-made reservoir dynamics that have as the steady solution the state to be protected [8, 9]. Another option is to use the possibility of accessing the information that leaks to the environment to simply post-select favourable events or to actively affect the system, as in the case of feedback methods [10]–[15]. A common feature of these procedures is the requirement for some sort of non-local interaction to counteract the deleterious effect of decoherence on entanglement. These collective interactions can be easily created when the systems to be entangled are close together, but they can become a challenge for systems that are far apart.

In the case of distant subsystems, the use of local strategies seems to be the most reasonable option from a practical point of view, although it is clear that any tactics based only on local operations would be useless for creating entanglement. However, if an initial entanglement is shared between the distant parties, one could hope that local actions could reduce or even stop entanglement decay.

Recently, local and non-local monitoring of quantum systems and its description in terms of quantum trajectories have been investigated in the context of entanglement decay [16]–[20] and local entanglement protection [21, 22]. In particular, Vogelsberger and Spehner [20] observed that the average entanglement could be preserved for systems coupled to local infinite temperature reservoirs under an optimal (in the sense that it maximizes the average entanglement) choice of a quantum trajectory unravelling.

In this paper, we present a proposal for the implementation of the above idea to preserve entanglement. Firstly, we derive the quantum dynamics conditioned on the measurement outcomes to show that the state evolves stochastically but remains entangled. Moreover, at any given time, the original state can be perfectly recovered as long as the detection signal is recorded with unity efficiency. Secondly, we show how to locally induce an artificial environment that reproduces the dynamics of decay and excitation reservoirs in order to mimic the infinite temperature condition. Finally, we provide a way to jointly, but locally, monitor

both the engineered and the natural reservoirs in terms of quantum jumps or homodyne-like measurements. In this way, we give a physical interpretation in terms of a measurement prescription for the unravelling leading to entanglement preservation. Note that the local character of the proposed scheme allows the application of it to an arbitrary number of qubits, which makes it useful for different sorts of quantum communication and computation protocols. In the end, we also discuss a specific experimental implementation of the scheme and analyse its robustness against imperfections.

2. Preserving entanglement through local reservoir measurements

We consider a composite system consisting of qubits that are weakly coupled to their own local and independent dissipative reservoirs at zero temperature with decay rates $\gamma_{-, \alpha}$. We also consider independently and locally engineered reservoirs that incoherently pump each subsystem with rates $\gamma_{+, \alpha}$. A practical way to produce these artificial environments will be presented in section 3. Under the Born and Markov approximations [23], the time evolution of the system is then given by the Lindblad [24] master equation,

$$\dot{\rho} = \sum_{i, \alpha} \gamma_{i, \alpha} \mathcal{D}[\sigma_{i, \alpha}] \rho, \quad (1)$$

where $\mathcal{D}[\sigma_{i, \alpha}] \rho = \sigma_{i, \alpha} \rho \sigma_{i, \alpha}^\dagger - (1/2)(\sigma_{i, \alpha}^\dagger \sigma_{i, \alpha} \rho + \rho \sigma_{i, \alpha}^\dagger \sigma_{i, \alpha})$, with $i = +, -$ and α indicating the subsystem on which the operators are acting. $\sigma_{-, \alpha}$ is the deexcitation operator for qubit α and $\sigma_{+, \alpha}$ its Hermitian conjugate. Note that in the particular case of $\gamma_+ \leq \gamma_-$, these engineered reservoirs simulate local thermal reservoirs of average numbers of photons $\bar{n} = \frac{\gamma_+}{\gamma_- - \gamma_+}$, and particularly of infinite temperature for $\gamma_- = \gamma_+$.⁵

These decoherence dynamics can be unravelled in terms of stochastic quantum trajectories in infinitely many ways, each one corresponding to a different scheme for continuously monitoring changes in the reservoirs that act on the qubits [25]–[29]. Quantum jumps, for example, describe the discrete clicks on a photodetector, while homodyne-like measurements are represented by continuous stochastic equations. In what follows, we will show how the local monitoring of (1) in terms of these kinds of trajectories can preserve the entanglement in the system.

2.1. Dynamics in terms of quantum jumps

The most common unravelling is given by choosing the jump operators as the ones appearing explicitly in (1), i.e. $J_{i, \alpha} = \sqrt{\gamma_{i, \alpha}} \sigma_{i, \alpha}$. The jump dynamics combine the action of randomly occurring quantum jumps given by the operators $J_{i, \alpha}$ with periods of no-jump evolution when the system evolves according to the operator $M_{\text{NJ}} = 1 - (1/2) \sum_{i, \alpha} J_{i, \alpha}^\dagger J_{i, \alpha}$. After the detection of a jump, the system evolves to the state $\rho_{J_{i, \alpha}}(t + dt) = J_{i, \alpha} \rho(t) J_{i, \alpha}^\dagger / \text{Tr}(J_{i, \alpha}^\dagger J_{i, \alpha} \rho(t))$, where the denominator gives the probability ($p_{i, \alpha}$) of detecting a jump i in the subsystem α . If no jump is detected in this interval, and assuming perfect detectors, the system then evolves to $\rho_{\text{NJ}}(t + dt) = M_{\text{NJ}} \rho(t) M_{\text{NJ}} / \text{Tr}(M_{\text{NJ}} \rho(t) M_{\text{NJ}})$. The continuous monitoring of the system then indicates the sequence of operators that have to be applied at each time step determining, in this way, the trajectory of the system conditioned on that specific measurement record.

⁵ The infinite temperature regime is to be understood as the limit where $\bar{n} \rightarrow \infty$ and $\gamma \rightarrow 0$ such that the rates γ_- and γ_+ are the same and remain finite. In this way, the system has equal chances of decaying or being excited.

In order to recover the unmonitored evolution of the density matrix given by the master equation, one has to average out all of the jump and no-jump possibilities for the qubits, $\rho(t + dt) = p_{\text{NJ}} \rho_{\text{NJ}}(t + dt) + \sum_{i,\alpha} p_{i,\alpha} \rho_{J_{i,\alpha}}(t + dt)$.

To analyse the state dynamics, first note that when $\gamma_{-, \alpha} = \gamma_{+, \alpha} = \gamma_{\alpha}$, the no-jump operator is proportional to the identity

$$M_{\text{NJ}} = 1 - \frac{dt}{2} \sum_{\alpha} \gamma_{\alpha} (\sigma_{-, \alpha} \sigma_{+, \alpha} + \sigma_{+, \alpha} \sigma_{-, \alpha}) = 1 - \sum_{\alpha} \frac{\gamma_{\alpha}}{2} dt. \quad (2)$$

In this particular case, M_{NJ} commutes with the jump operators $J_{i,\alpha}$, which means that the moment in which jumps are applied does not influence the final state of the system, but influences only the number and type of jumps. In other words, the un-normalized conditional state of the system at time t for a given trajectory will be given by the sequential application of jumps to the initial state,

$$\rho_c(t) = \left(\prod_s^N J_s \right) \rho(0) \left(\prod_s^N J_s \right)^{\dagger}, \quad (3)$$

where, once again, each J_s corresponds to one of the possible $J_{i,\alpha}$ described above, and N is the number of jumps detected in the particular trajectory.

Also note that any of these jumps immediately destroys all of the entanglement originally present in the system, since it identifies the state of the subsystem whose reservoir ‘clicked’. For example, if a spontaneously emitted photon is detected in the reservoir acting on a particular qubit (let us call it A), then the jump $J_{-,A}$ projects A into its ground state $|0\rangle$. Therefore, this choice of monitoring scheme only allows the preservation of entanglement for trajectories where no jumps are detected, which happens with an exponentially decreasing probability in time. Therefore, the averaged entanglement over all possible trajectories also decays, even though it may still last longer than the entanglement of the unmonitored evolution [19].

However, as mentioned before, this choice of jumps is not unique. In fact, new jumps defined as $\tilde{J}_k = \sum_i U_{ki} J_i$, with U (left) unitary, leave the master equation unaltered⁶ while generating a distinct set of trajectories. Observe that the no-jump trajectory remains the same, independent of the unitary transformation U and therefore the conditioned state at time t can still be found by the sequential application of the detected jump operators.

We will now show that a particular choice of jump operators can in fact preserve the entanglement in every trajectory. If instead of $\sigma_{\pm, \alpha}$ we choose as new jumps the linear combinations

$$J_{x,\alpha} = \sqrt{\frac{\gamma_{\alpha} dt}{2}} (\sigma_{-, \alpha} + \sigma_{+, \alpha}) = \sqrt{\frac{\gamma_{\alpha} dt}{2}} \sigma_{x,\alpha}, \quad (4a)$$

$$J_{y,\alpha} = i \sqrt{\frac{\gamma_{\alpha} dt}{2}} (\sigma_{-, \alpha} - \sigma_{+, \alpha}) = \sqrt{\frac{\gamma_{\alpha} dt}{2}} \sigma_{y,\alpha}, \quad (4b)$$

⁶ This unitary is not the most general transformation on the jumps that leave the master equation invariant. Addition of a complex number to the jumps with a corresponding change in the effective Hamiltonian is also possible [29], but we omitted it here for simplicity.

the jump operations will now be local and unitary; hence they cannot change the entanglement shared by the qubits. Since the product of Pauli operators is again a Pauli operator or the identity, the conditioned state $\rho_c(t)$ of the system at time t in any given trajectory will be given by

$$\rho_c(t) = \bigotimes_{\alpha} \sigma_{\text{Pauli},\alpha} \rho(0) \bigotimes_{\alpha} \sigma_{\text{Pauli},\alpha}, \quad (5)$$

where each $\sigma_{\text{Pauli},\alpha} = \prod_s^N J_{s,\alpha}$ is one of $\{\mathbb{1}, \sigma_{x,\alpha}, \sigma_{y,\alpha}, \sigma_{z,\alpha}\}$ and is determined by the sequence of jumps recorded on the respective subsystem α . In each trajectory, entanglement is preserved because the system randomly jumps from one entangled state to another. However, provided that the parts keep track of their sequence of monitored jumps, the initial state can be recovered by locally reversing the final Pauli operation conditioned on the measurement results. Note that the scheme is valid for any number of qubits since it relies only on the fact that the jumps are local unitary operations and the state does not change between jumps.

2.2. Dynamics in terms of diffusive unravellings

The same idea can be applied to continuous in time stochastic unravelling. For perfect detection efficiency, the evolution of the system is given by [27]–[29]

$$d\rho_{\vec{Y}} = \sum_{i,\alpha} \gamma_{\alpha} \mathcal{D}[\sigma_{i,\alpha}] \rho_{\vec{Y}} dt + \sum_{i,\alpha} \sqrt{\gamma_{\alpha}} [(\sigma_{i,\alpha} - \langle \sigma_{i,\alpha} \rangle) \rho_{\vec{Y}} d\xi_{i,\alpha}^* + \text{h.c.}], \quad (6)$$

where $d\xi_{i,\alpha}$ are complex Wiener increments satisfying

$$d\xi_{i,\alpha}(t) d\xi_{j,\alpha'}^*(t) = dt \delta_{ij} \delta_{\alpha\alpha'}, \quad (7)$$

$$d\xi_{i,\alpha}(t) d\xi_{j,\alpha'}^*(t) = dt u_{ij,\alpha} \delta_{\alpha\alpha'}, \quad (8)$$

with u being a complex symmetric matrix subject to the condition $\|u\|_2 \leq 1$ for the matrix two-norm. Note that the $\delta_{\alpha\alpha'}$ indicates that all measurements are local and no correlation between the noises in the different subsystems is allowed in our model. The subscript \vec{Y} in ρ indicates that its evolution is conditioned on the measurement record given by the complex currents

$$Y_{i,\alpha} dt = \sum_j dt \sqrt{\gamma_{\alpha}} \langle \sigma_{i,\alpha} + u_{ij,\alpha} \sigma_{j,\alpha}^{\dagger} \rangle + d\xi_{i,\alpha}. \quad (9)$$

This time, the freedom in the choice of unravelling lies in the values of the noise correlations encoded in the matrix u . Each different u is associated with generalized homodyne-like measurements and corresponds to a particular way of mixing the signals from the monitored system and the noise introduced by external classical sources. To find the value that maximizes the average entanglement, one can derive directly an equation for the entanglement in the system [30], as has been done for the case of the zero temperature reservoir [18, 20]. In what follows, we will show that the choice $u_{ij,\alpha} = -1$ for $i \neq j$ and $u_{ii,\alpha} = 0$ leads to perfect entanglement protection for a diffusive monitoring.

To see that, first note that with this choice of u the complex noises can be written as

$$d\xi_{-,\alpha} = \frac{dW_{1,\alpha} + idW_{2,\alpha}}{\sqrt{2}}, \quad (10)$$

$$d\xi_{+,\alpha} = \frac{-dW_{1,\alpha} + idW_{2,\alpha}}{\sqrt{2}}, \quad (11)$$

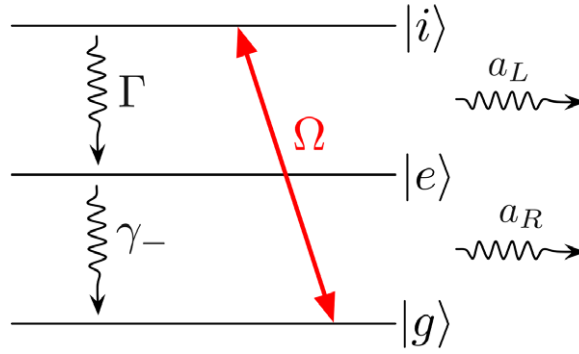


Figure 1. Level scheme for engineering the reservoir. The qubit is encoded in levels $|g\rangle$ and $|e\rangle$, which can spontaneously decay back to $|g\rangle$. A classical field couples $|e\rangle$ and $|i\rangle$, which decays back to $|e\rangle$. When the decay rate Γ is much larger than the other frequencies of the problem, the level $|i\rangle$ can be eliminated and an effective artificial excitation reservoir for the qubit is created.

with the real noises dW obeying $dW_{i,\alpha}(t)dW_{j,\alpha'}(t) = dt\delta_{ij}\delta_{\alpha\alpha'}$. Using these relations and $\sigma_{-,\alpha} = (\sigma_{x,\alpha} - i\sigma_{y,\alpha})/2$, $\sigma_{+,\alpha} = (\sigma_{x,\alpha} + i\sigma_{y,\alpha})/2$, we can express (6) in a more convenient form,

$$d\rho_{\bar{Y}} = \sum_{\alpha} \frac{\gamma_{\alpha}}{2} dt (\sigma_{x,\alpha} \rho_{\bar{Y}} \sigma_{x,\alpha} + \sigma_{y,\alpha} \rho_{\bar{Y}} \sigma_{y,\alpha} - 2\rho_{\bar{Y}}) - i \sum_{\alpha} \sqrt{\frac{\gamma_{\alpha}}{2}} [(dW_{1,\alpha} \sigma_{y,\alpha} + dW_{2,\alpha} \sigma_{x,\alpha}), \rho_{\bar{Y}}]. \quad (12)$$

Now it is easy to show that this evolution can be mapped into the application of local stochastic Hamiltonians defined as $H_{\alpha} = \sqrt{\gamma_{\alpha}}(dW_{1,\alpha}\sigma_y + dW_{2,\alpha}\sigma_x)/\sqrt{2}$. The conditional state of the system at $t + dt$ can be written as

$$\begin{aligned} \rho_{\bar{Y}}(t + dt) &= e^{-iH_{\alpha}} \rho_{\bar{Y}}(t) e^{iH_{\alpha}} \\ &= \left(1 - iH_{\alpha} - \frac{H_{\alpha}^2}{2}\right) \rho(t) \left(1 + iH_{\alpha} - \frac{H_{\alpha}^2}{2}\right), \end{aligned} \quad (13)$$

where in the second step we expanded the exponentials up to first order in dt . Replacing the definition of H_{α} and using the correlation properties of the real noises dW , one obtains back (12). The evolution of the system therefore is equivalent to the application of a sequence of local Hamiltonians to the system, hence unitary and local. Once again, the state evolves stochastically from one entangled state to another, now in a diffusive way, and entanglement is preserved for each trajectory. Moreover, because each subsystem has the record of their current signals and the local unitaries are state independent, whenever each party decides to use the qubits for any application, they just need to locally apply a final unitary matrix to revert the system back to its initial state.

3. Engineering and monitoring the environment

We proceed to show the engineering of the artificial temperature reservoir in atomic-like systems (figure 1). Assume that each qubit corresponds to the two lower levels of a three-level atom, $|0\rangle = |g\rangle$ and $|1\rangle = |e\rangle$, and that level $|e\rangle$ spontaneously decays into level $|g\rangle$ at a rate γ_{-} , emitting a photon circularly polarized to the right (annihilation operator a_R). The detection

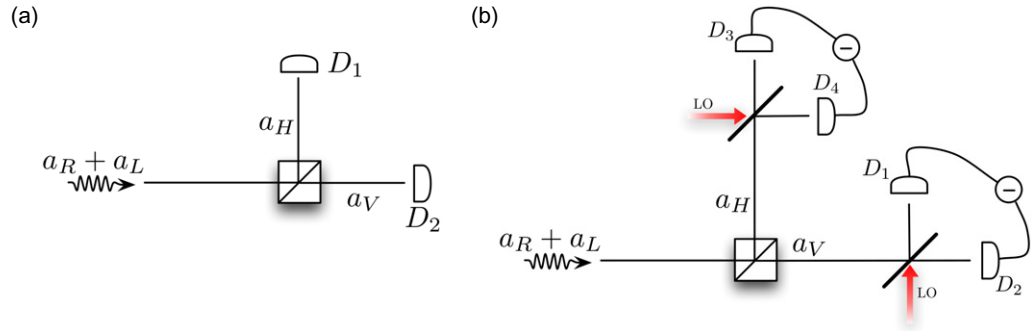


Figure 2. Monitoring scheme: photons corresponding to decay and excitation processes are collected and passed through a PBS. For the quantum jump case (a), the detection of photons after the PBS will project the atomic state according to the optimal jump unravelling. For the diffusive case (b), the photons emerging from the PBS are combined with local oscillators and the signals are recorded, providing the currents in (14a) and (14b).

of this photon corresponds to a $J_- \propto \sigma_{ge}$ jump and upon detection of it the state of the atom is certainly level $|g\rangle$. Let us also assume that levels $|g\rangle$ and $|i\rangle$ have the same m_z projection of angular momentum and that an external π -polarized classical field of intensity Ω is turned on at the $|g\rangle \rightarrow |i\rangle$ frequency. In this case, if $\Omega \ll \Gamma$, then level $|i\rangle$ can be adiabatically eliminated and the overall effect of turning on the external field is to incoherently pump level $|g\rangle$ back to level $|e\rangle$ at a rate given by $\gamma_+ = \frac{4\Omega^2}{\Gamma}$ and with an emission of a photon circularly polarized to the left (annihilation operator a_L). Now, the detection of this photon corresponds to a $J_+ \propto \sigma_{eg}$ jump and upon detection of it the state of the atom is certainly level $|e\rangle$. Note that if the same scheme is applied to all subsystems and the emitted photons are ignored, the dynamics of the qubits are described exactly by (1). Furthermore, if $\frac{4\Omega^2}{\Gamma} = \gamma_-$, then the incoherent pump, together with the spontaneous decay, simulates infinite temperature for the photonic reservoir.

Within this model, it is now easy to describe the setup for implementing the entanglement-preserving monitoring. For each subsystem, the scheme shown in figure 2(a) has to be applied. If both photons have the same energy⁷ and polarized beam splitters (PBS) are placed before the detectors, the photons will be combined in linear orthogonal polarizations $a_H = (a_R + a_L)/\sqrt{2}$ and $a_V = i(a_L - a_R)/\sqrt{2}$. Their detection after the PBS will then correspond to the optimal jump operators given in (4a) and (4b).

For diffusive monitoring, the scheme is shown in figure 2(b). After the PBS, the photons are combined with local oscillators at beam splitters and a homodyne measurement is carried out. The currents obtained by subtracting the signals at the pairs of detectors D_1 and D_2 , and D_3 and D_4 , are

$$I_{1-2} dt = \sqrt{\frac{\gamma}{2}} \langle (a_R - a_L^\dagger) - (a_L - a_R^\dagger) \rangle dt + dW_1, \quad (14a)$$

$$I_{3-4} dt = -i\sqrt{\frac{\gamma}{2}} \langle (a_R - a_L^\dagger) + (a_L - a_R^\dagger) \rangle dt + dW_2, \quad (14b)$$

⁷ If $\omega_{ie} - \omega_{eg} = \Delta$, then external fields can be applied to shift the electronic levels to produce photons of the same energy.

with dW_i being real independent increments associated with the noise in the detectors. Now using the fact that the detection of photons a_R and a_L corresponds, respectively, to the action of σ_- and σ_+ in the atomic system, one can combine the above signals to obtain complex currents for the entanglement protecting unravelling, namely $Y_- = I_{1-2} + iI_{3-4}$ and $Y_+ = -I_{1-2} + iI_{3-4}$. Note that due to the symmetry of the reservoir, when one makes the correspondences $a_R \rightarrow \sigma_-$ and $a_L \rightarrow \sigma_+$, the average value terms in the signals vanish, indicating that for the entanglement protecting monitoring the signals contain only the noise parts.

From a practical point of view, the difficult part in the above scheme can be the detection of spontaneously emitted photons due to the broad solid angle of emission. For this reason, the scheme would be more suited to implementing in one-dimensional (1D) systems where the emitters are embedded in broadband waveguides that allow the propagation of light in only one direction. Such systems are realizable nowadays in different experimental setups, such as nanowires [31, 32] and superconducting circuit QED [33, 34].

4. Analysis of imperfections

In all the schemes presented here, the biggest problem is the detection inefficiency, especially because the added reservoir increases the emission rates in each qubit, therefore increasing the effect of the decoherence mechanism. We now proceed to calculate the effect of inefficiencies in our proposal. Inefficiency can be treated by including a η parameter in the master equation, such that $\eta = 1$ means perfect detectors, and by rewriting (1) as

$$\dot{\rho} = \eta \sum_{i,\alpha} \gamma_{i,\alpha} \mathcal{D}[\sigma_{i,\alpha}] \rho + (1 - \eta) \sum_{i,\alpha} \gamma_{i,\alpha} \mathcal{D}[\sigma_{i,\alpha}] \rho. \quad (15)$$

The part proportional to η corresponds to the one about which we can obtain information and therefore the one we can unravel. The remaining part corresponds to a decoherence evolution, ‘slowed’ by a factor $(1 - \eta)$, due to the information that is flowing to the reservoir and not being retrieved.

We shall now analyse the effect of detection inefficiencies in the quantum jump description. The most favourable unravelling described in section 2.1 has a no-jump proportional to identity and jumps proportional to σ_x and σ_y . The difference now is that one has to add to the no-jump evolution a term corresponding to the decoherence part proportional to $(1 - \eta)$. Despite this difference, the analysis remains basically unaltered since the new no-jump evolution still commutes with the jump operations. It suffices to check this for one of the qubits since operations in different subsystems trivially commute. If one writes the local decoherence part in terms of σ_x and σ_y as in (12), one can easily verify that the action of a jump followed by decoherence ($\mathcal{D}[\sigma_x](\sigma_i \rho \sigma_i) + \mathcal{D}[\sigma_y](\sigma_i \rho \sigma_i)$) coincides with the evolution with decoherence followed by the action of a jump ($\sigma_i (\mathcal{D}[\sigma_x] \rho + \mathcal{D}[\sigma_y] \rho) \sigma_i$), where i is either x or y .

As a result, the evolution of the density matrix of the system given that N jumps were detected will be given by the mixed state version of (5),

$$\rho_c(t) = \bigotimes_{\alpha} \sigma_{\text{Pauli},\alpha} \rho(t) \bigotimes_{\alpha} \sigma_{\text{Pauli},\alpha}, \quad (16)$$

where $\rho(t)$ is given by the integration of the equation $\dot{\rho} = (1 - \eta) \sum_{i,\alpha} \gamma_{i,\alpha} \mathcal{D}[\sigma_{i,\alpha}] \rho$. Note that this equation is equivalent to the unmonitored one except for the final application of local unitaries (that can be reversed since the parties know all the detected jumps), and the factor

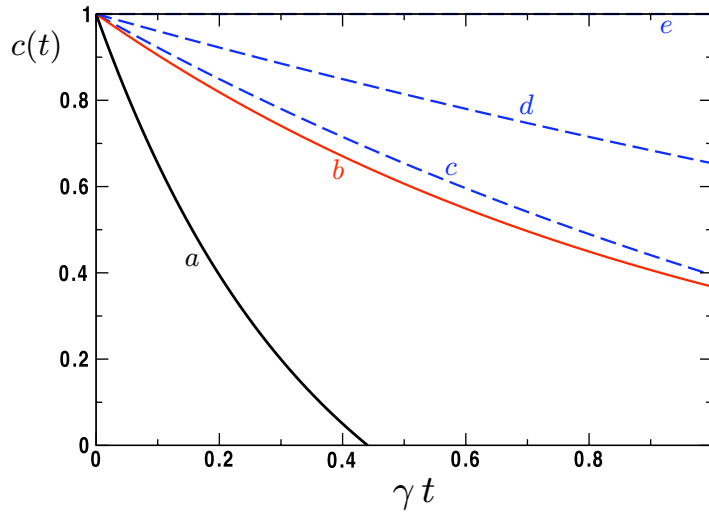


Figure 3. Concurrence as a function of time for unmonitored reservoirs at infinite (a) and zero (b) temperatures. The dashed curves represent the average concurrence for the engineered reservoir under monitoring for $\eta = 0.8$ (c), 0.9 (d) and 1.0 (e).

$(1 - \eta)$ corresponding to the detection efficiency. As previously noted, if the detectors are perfect, entanglement is fully preserved. Otherwise, the detection inefficiency becomes simply a multiplier factor on the decoherence rate.

We are now in a position to compare our scheme with both the monitored dynamics without the engineered reservoir and the unmonitored evolution. For that we consider the particular case of two qubits, A and B , that share initially a maximally entangled state of the form $|\psi_0\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$. We also assume that the decay rates are the same ($\gamma_A = \gamma_B = \gamma$). For this state, the unconditional entanglement, as measured by concurrence [35], decays as $c(t) = e^{-\gamma t}$ and $c(t) = e^{-2\gamma t} + (e^{-4\gamma t}/2) - (1/2)$ for zero and infinite temperature reservoirs, respectively [36]. The corresponding curves are shown by the solid lines in figure 3: black (a) for T_∞ and red (b) for T_0 . At zero temperature, any local monitoring with $\eta = 1$ gives an average entanglement of $e^{-\gamma t}$ [17, 18], coinciding with the master equation result (b). The concurrence for the entanglement conditioned on jump monitoring including the engineered reservoir can be calculated using (16) and gives $c(t) = e^{-2\gamma(1-\eta)t} + (e^{-4\gamma(1-\eta)t}/2) - (1/2)$. The dashed blue curves (c) and (d) represent the cases with $\eta = 0.8$ and $\eta = 0.9$, respectively, and show that, for the time scale shown in the figure, monitoring preserves entanglement better than not adding the artificial reservoir. For any $\eta < 1$, the dynamics will always have an infinite temperature component (with rate $\gamma(1 - \eta)$) and the solution [36] will lead to disentanglement of the system at a finite time [37], which does not occur for initial Bell states at zero temperature. This means that the curves for the monitored system with an artificial reservoir will have to cross the curve of the unmonitored system for the original zero temperature bath. The higher the detection efficiency, the longer our scheme will be advantageous as compared to the unconditional evolution. Obviously, for $\eta = 1$ (curve e), one has perfect protection for all times. Although this analysis has been presented for the case of two qubits, we should emphasize that it is also valid for multipartite systems, with perfect protection occurring for ideal monitoring and decline in performance as the detection inefficiency increases.

For a diffusive unravelling, the inefficient evolution is not simple to calculate as in the jump case, because now the sequence of evolutions do not commute with the unmonitored evolution. However, we can still numerically simulate the equations and the result obtained follows exactly the curves for the jump monitoring. Therefore, since both kinds of monitoring perform equally well theoretically, choosing one monitoring scheme over the other is a matter of finding the one that can be implemented with the best possible detection efficiency.

Finally, if the reservoirs have natural temperature, then the incoherent pump needs to be weaker in order to balance out both reservoirs. The equation, in this case, will be similar to the one with inefficiencies, with the natural excitation reservoir, which is not monitored, playing a role similar to that of an additional inefficiency to the detection scheme.

In conclusion, we have shown a way to engineer an artificial reservoir that increases the temperature of the system and yet can be used to decrease the rate of entanglement decay, or even stop it completely, when the system is properly (locally) monitored. With the use of simple optical elements, we showed how the monitoring scheme that preserves entanglement can be implemented and we derived a way to recover the initial state from the measurement record using only local unitary operations. In this way, the loss of information from the system due to the interaction with the environment (which, when irreversible, is ultimately the poison responsible for the decoherence process itself) can also be the vaccine used to fight decoherence as long as the information can be retrieved by a suitable monitoring scheme.

Acknowledgments

We acknowledge support from the Center for Quantum Technologies at the National University of Singapore. ARRC acknowledges financial support from the Australian Research Council Centre of Excellence program and MFS acknowledges support from the National Research Foundation and the Ministry of Education of Singapore.

References

- [1] Shor P W 1995 Scheme for reducing decoherence in quantum computer memory *Phys. Rev. A* **52** R2493–6
- [2] Calderbank A R and Shor P W 1996 Good quantum error-correcting codes exist *Phys. Rev. A* **54** 1098–105
- [3] Ekert A and Macchiavello C 1996 Quantum error correction for communication *Phys. Rev. Lett.* **77** 2585–8
- [4] Gottesman D 1996 Class of quantum error-correcting codes saturating the quantum hamming bound *Phys. Rev. A* **54** 1862–8
- [5] Viola L, Knill E and Lloyd S 1999 Dynamical decoupling of open quantum systems *Phys. Rev. Lett.* **82** 2417–21
- [6] Zanardi P and Rasetti M 1997 Noiseless quantum codes *Phys. Rev. Lett.* **79** 3306–9
- [7] Lidar D A, Chuang I L and Whaley K B 1998 Decoherence-free subspaces for quantum computation *Phys. Rev. Lett.* **81** 2594–7
- [8] Poyatos J F, Cirac J I and Zoller P 1996 *Phys. Rev. Lett.* **77** 4728
- [9] Carvalho A R R, Milman P, de Matos Filho R L and Davidovich L 2001 Decoherence pointer engineering and state protection *Phys. Rev. Lett.* **86** 4988
- [10] Stockton J K, van Handel R and Mabuchi H 2004 Deterministic Dicke-state preparation with continuous measurement and control *Phys. Rev. A* **70** 022106
- [11] Mancini S and Wang J 2005 Towards feedback control of entanglement *Eur. Phys. J. D* **32** 257
- [12] Mancini S 2006 Markovian feedback to control continuous-variable entanglement *Phys. Rev. A* **73** 010304

- [13] Mancini S and Wiseman H M 2007 Optimal control of entanglement via quantum feedback *Phys. Rev. A* **75** 012330
- [14] Wang J, Wiseman H M and Milburn G J 2005 Dynamical creation of entanglement by homodyne-mediated feedback *Phys. Rev. A* **71** 042309
- [15] Carvalho A R R and Hope J J 2007 Stabilizing entanglement by quantum-jump-based feedback *Phys. Rev. A* **76** 010301
- [16] Nha H and Carmichael H J 2004 Entanglement within the quantum trajectory description of open quantum systems *Phys. Rev. Lett.* **93** 120408
- [17] Carvalho A R R, Busse M, Brodier O, Viviescas C and Buchleitner A 2007 Optimal dynamical characterization of entanglement *Phys. Rev. Lett.* **98** 190501
- [18] Viviescas C, Guevara I, Carvalho A R R, Busse M and Buchleitner A 2010 Entanglement dynamics in open two-qubit systems via diffusive quantum trajectories *Phys. Rev. Lett.* **105** 210502
- [19] Mascarenhas E, Cavalcanti D, Vedral V and Santos M F 2010 Physically realizable entanglement by local continuous measurements [arXiv:1006.1233](https://arxiv.org/abs/1006.1233)
- [20] Vogelsberger S and Spehner D 2010 Average entanglement for Markovian quantum trajectories *Phys. Rev. A* **82** 052327
- [21] Mascarenhas E, Marques B, Cavalcanti D, Cunha T M and Santos F M 2010 Protection of quantum information and optimal singlet conversion through higher-dimensional quantum systems and environment monitoring *Phys. Rev. A* **81** 032310
- [22] Mascarenhas E, Marques B, Cunha M T and Santos M F 2010 Continuous quantum error correction through local operations *Phys. Rev. A* **82** 032327
- [23] Breuer H P and Petruccione F 2002 *The Theory of Open Quantum Systems* (Oxford: Oxford University Press)
- [24] Lindblad G 1976 On the generators of quantum dynamical semigroups *Math. Phys.* **48** 119
- [25] Carmichael H 1993 *An Open Systems Approach to Quantum Optics (Lecture Notes in Physics m18)* (Berlin: Springer)
- [26] Mølmer K, Castin Y and Dalibard J 1993 Monte Carlo wave function method in quantum optics *J. Opt. Soc. Am. B* **10** 524
- [27] Rigo M, Mota-Furtado F and O'Mahony P F 1997 Continuous stochastic Schrödinger equations and localization *J. Phys. A: Math. Gen.* **30** 7557
- [28] Wiseman H M and Diósi L 2000 Complete parameterization, and invariance, of diffusive quantum trajectories for Markovian open systems *Chem. Phys.* **268** 91–104
- [29] Wiseman H M and Milburn G J 2010 *Quantum Measurement and Control* (Cambridge: Cambridge University Press)
- [30] Guevara I *et al* unpublished work
- [31] Chang D E, Sorensen A S, Demler E A and Lukin M D 2007 A single-photon transistor using nanoscale surface plasmons *Nat. Phys.* **3** 807–12
- [32] Claudon J, Bleuse J, Malik N S, Bazin M, Jaffrennou P, Gregersen N, Sauvan C, Lalanne P and Gerard J-M 2010 A highly efficient single-photon source based on a quantum dot in a photonic nanowire *Nat. Photon.* **4** 174–7
- [33] Houck A A *et al* 2007 Generating single microwave photons in a circuit *Nature* **449** 328–31
- [34] Mallet F, Ong F R, Palacios-Laloy A, Nguyen F, Bertet P, Vion D and Esteve D 2009 Single-shot qubit readout in circuit quantum electrodynamics *Nat. Phys.* **5** 791–5
- [35] Wootters W K 1998 Entanglement of formation of an arbitrary state of two qubits *Phys. Rev. Lett.* **80** 2245
- [36] Carvalho A R R, Mintert F, Palzer S and Buchleitner A 2007 Entanglement dynamics under decoherence: from qubits to qudits *Eur. Phys. J. D* **41** 425–32
- [37] Diósi L 2003 Progressive decoherence and total environmental disentanglement ed F Benatti and R Floreanini *Irreversible Quantum Dynamics (Lecture Notes in Physics vol 622)* (Berlin: Springer) pp 157–63