Developing Student Understanding of Place Value and Supporting Teachers' Confidence in Teaching Place Value via a Digital Tool

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Developing student understanding of place value and supporting teachers' confidence in teaching place value via a digital tool

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This article describes Ulrich Kortenkamp's Place Value Chart app and the accompanying teacher guide developed by the members of an international research team led by regular contributor Kevin Larkin.

Editor's Note: The authors acknowledge that the use of the Kortenkamp Place Value App may represent a potential or perceived conflict of interest. I leave it to the reader to form their own opinion of the app and/or article based on its own merits. Please note that this was an editorial decision and should not be considered an endorsement by APMC or AAMT.

Introduction

This article builds upon earlier work by mathematics colleagues (e.g. Bailey, 2015; Hartnett, 2018; Rogers, 2012) published in the APMC journal regarding the fundamental importance of place value (PV) understanding. In this paper we explore the utility of an iPad app, Place Value Chart, which was custom-designed to support the development of student PV understanding across all primary school year levels. This article also describes an international collaborative effort to support teachers' Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) through the creation of a 25-page teachers' guide outlining the use of the app.

Importance of place value

The decimal number system is a powerful tool for writing mathematics and doing arithmetic as any rational number can be written using only ten

different digits in a unique way (Kortenkamp & Ladel, 2014). The importance of a highly integrated understanding of place value is acknowledged across a wide spectrum of student learning scenarios (Berman, 2011). Place value involves the learning of a number of "big ideas" (outlined below) and fundamental to these is the integration of early experiences of counting by ones and grouping numbers in tens; and later linking these groupings to the corresponding written numeral and how numbers are said (Bailey, 2015).

Given the broad sweep of concepts included within PV, it is perhaps unsurprising that an integrated understanding of PV is problematic for some students. Rogers (2012) notes that "despite the unchanging and recursive nature of our base-ten system, it seems some students never manage to fully unravel the hidden code that underlies place value" (p. 648). Rogers also suggests that although students may appear to understand some elements of PV, their explanations of place value are often "trivial or glib and not representative of their knowledge" (p. 649). Kortenkamp and Ladel (2014) reinforce this view, identifying that PV "is difficult to understand and to teach" (p. 35). Of particular concern is a full understanding of decimal fractions and their representations within our PV system. Roche (2010) cites a significant body of research indicating the prevalence and persistence of misconceptions relating to decimal fractions; and Rogers (2012) argues that students' "weak grasp of decimal fractions can be attributed to their shaky foundation in whole number place value" (p. 648).

Due to space constraints, we have only highlighted a small component of the wide body of research on PV learning by students. It is clear, however, that support for students (and teachers) in relation to PV continues to be timely. We begin by very briefly outlining the big ideas of PV.

Big ideas of place-value

A range of mathematics researchers have proposed various properties of our PV system and then outlined various sub-concepts that underpin these properties. Whilst differing slightly in emphasis, the following is generally agreed.

The decimal place value system is based on five interwoven properties (see Behrens, 2015; Kortenkamp & Ladel, 2013):

1. **Property of continued bundling**—if there are ten or more of the same bundle, we make a new bundle, until there are less than ten of any bundle type
2. **Base-ten property**—the values of the positions increase in powers of ten from right to left and decrease in powers of ten from left to right
3. **Positional property**—the value of the individual digits is determined by their position
4. **Multiplicative property**—the value of an individual digit (i.e., its place value) is found by multiplying the value of the digit (i.e., its face value) by the value assigned to its position
5. **Additive property**—the quantity represented by the whole numeral is the sum of the values represented by the individual digits.

There is also broad agreement that the following concepts are needed for a full understanding of PV (see Hartnett, 2018; Kortenkamp & Ladel, 2013; Rogers, 2012):

- Counting forwards and backwards in place value parts (e.g., 145, 245, 345 is counting using the unit hundred)
- Make, represent or identify the value of a number using a range of materials or models (e.g., Base ten blocks, Montessori Place Value Cards)
- Read and write a number in words and figures (e.g., 53 is written as 'fifty-three') and identify the place value of digits in a number (e.g., the place value of 3 in 234 is three tens or thirty)
- Recognise and complete partitions and regrouping of numbers (e.g., 6210 has 621 tens or 6210 ones)

- Compare numbers to determine which is larger or smaller and place them in descending or ascending order
- Apply place value knowledge when completing calculations (e.g., 140 add 50 is 14 tens add 5 tens)
- Use knowledge of magnitude of numbers when estimating (e.g., estimate how many seconds old a person is: 1000? 100 000? 10 000 000?)
- Transfer the different kinds of partition (i.e., standard partition, non-standard partition strong, non-standard partition weak, e.g., 1H 4T 5O = 1H 45O = 1H 2T 25O etc.)

Teachers can support students in developing a sound understanding of PV by focussing on a number of mathematical ideas that underpin the properties and concepts noted above. Firstly, students need to be familiar with the concept of the unit and recognise the difference between units of ten and units of one as this thinking encourages a transition from thinking about numbers as units to thinking of them as comprised of composite units (Bicknell, Young-Loveridge & Simpson, 2017). Secondly, students need to understand the idea of bundling, as this is a key factor in the uniqueness of the decimal system. As Kortenkamp and Ladel (2013) indicate "we can bundle a cardinal representation of a number by bundling in tens, and tens of tens, and tens of tens of tens, … until no further bundling is possible. This process will always lead to the same number of (less than ten for each place) bundles" (p. 189). Once children develop an understanding of bundling, they will likely see the connection between the part-whole concept and creating bundles and this becomes important in later computation work, for example, where they will be required to replace a bundle of ten objects with ten single objects in order to complete subtraction activities.

The role of materials in supporting the development of PV understanding

The necessity of using a variety of physical materials in building PV understanding should not be underestimated (Bailey, 2015). Previous articles in APMC have outlined a range of physical resources that might be helpful in teaching various aspects of PV. These examples include Multi-base Arithmetic Blocks (MAB) (Hartnett, 2018), Bundling Sticks (Broadbent, 2004), Decimats (Roche, 2010), Counting Cards (Taylor, 2004), Ten Frames (Hartnett, 2018) and Place Value Pac Man Cards (Russo, 2018). Given that each of the resources above are useful at different stages of

the development of PV understanding, Larkin (2016) provided a schematic to assist teachers in the "how, when and why" of selecting and then using physical materials in mathematics classrooms. Before explaining in some detail the Place Value Chart app, we discuss the affordances of digital manipulatives in mathematics classrooms.

While not all apps are thoughtfully designed, at their core is the significant potential of digital manipulatives is to provide students with opportunities to interweave pictorial and symbolic representations, with the actions that they perform on them, to emphasise the underlying mathematical concepts. The positive outcomes attributed to the use of physical manipulatives are also evident from research. Larkin, Kortenkamp, Ladel and Etzold (2019) cited a range of research that found that student ideas about shapes were more precise and mathematically robust after using computer-based Logo software and that affective gains followed student use of virtual manipulatives due to the immediate feedback which enhanced student self-efficacy. Given the outcomes of recent research on digital manipulatives, we suggest that it is broadly uncontested that they play an important role in primary mathematics education. Although these findings predate the relatively recent introduction of tablet apps, the added interactivity of tablet devices will likely improve student outcomes (Larkin et al., 2019).

When using digital manipulatives in mathematics classrooms, teachers should always consider whether they support the students' learning process in ways that physical materials do not allow (Behrens, 2015; Larkin, 2016), for example automatically bundling a set of ten units into one ten unit. As Hartnett (2018) notes, "the use of materials to support place-value learning needs careful planning and teachers cannot assume that students are making sense of the representations the same way as the teacher" (Hartnett, 2018, p.38). Whilst acknowledging these important provisos, we suggest that the Place Value Chart app is an appropriate digital resource for teaching the "big ideas" of PV and which offers learning and teaching advantages not present in the physical materials already discussed in previous APMC publications.

Figure 1. Place Value Chart. Available from https://itunes.apple.com/au/app/place-value-chart/id568750442?mt=8

Place value chart: A digital manipulative for teaching place value

Here we argue that the use of the Place Value Chart app complements non-digital mathematics activities already taking place.

1. What the app does

Place Value Chart combines the principle of bundling with the principle of place value. Via touching the screen with fingers (multi-touch), the children can represent numbers in the place value chart e.g., 241 as 2H, 4T and 1O (see Figure 2) and then later represent the same amount as 1H 14T 1O (see Figure 3).

Figure 2. Standard representation of 241 (2H 4T 1O).

But there are situations such as the written algorithms for, in this instance subtraction, where other kinds of partition are needed e.g., you cannot easily subtract 63 (i.e., 6T and 3O) from 241, as there are only 4T and 1O. The children first need to unbundle 1H.

Figure 3. Non-Standard representation of 241(1H 14T 1O).

Now there are 14T and you can subtract 6T, which leave 8T.

Figure 4. Standard representation of 181 (1H 8T 1O).

To subtract 3O the children now need to unbundle 1T.
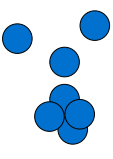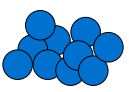

181		
1 Hundred	7 Tens	11 Ones

Figure 5. Non-Standard representation of 181 (1H 7T 110).

Now there are 110 and it is possible to subtract the 3O leaving 8O.
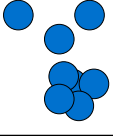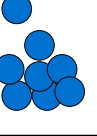

178		
1 Hundred	7 Tens	8 Ones

Figure 6. Standard representation of 178 (1H 7T 80).

So what the app does, in the case of subtraction, is automatically unbundle a group of ten dots if the child moves a token from one place (column) to the place immediately to the right. In that way, the value of number remains the same (241 = 2H 4T 1O = 1H 14T 1O= 1H 13T 11O), with only the representation changing. The app also automatically bundles ten dots into one dot on moving one dot one column to the left (on the proviso that there are at least ten dots in the initial column prior to the move). The app can also bundle and unbundle in 100s and 1000s (and later tenths and hundredths) if the minimum number of dots required for the move is available (for the bundling component).

Bundling and unbundling within the place value chart is very important. For example, children benefit from the Strong Nonstandard Partitioning (Ladel & Kortenkamp, 2014) when solving tasks using division. If they recognise that there are multiples of the divisor in the dividend, they can easily divide the whole number e.g., $16\ 832 \div 4$.
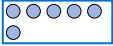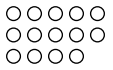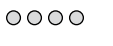
	TTh	Th	H	T	O			TTh	Th	H	T	O
	1	6	8	3	2	÷4=						
		16	8		32	÷4=				4	2	8

$16\ 832 \div 4 = 16\text{Th } 8\text{H } 32\text{O} \div 4 = 4\text{Th } 2\text{H } 8\text{O} = 4208$. In that way it is not necessary to complete the written division algorithm; however, it is necessary to have number sense "to see" the multiples of the divisor.

The (not strong) Nonstandard Partitioning is needed to understand all of the written algorithms (i.e., calculation methods). For example in the written addition algorithm with regrouping, it is an intermediate step to have more than nine objects in a column. To write the result (sum) in the example (see Figure 7) without the place value chart, you have to bundle the ten Ones to one Tens, and to add it to the already existing eight Tens.

Hence, the Place Value Chart app provides an opportunity to visualise the written algorithm and to operate on the enactive form of representation. This applies not only for the addition, but also for subtraction, multiplication and division.

2. How the app works

There is a range of settings that the teacher may change depending on the children's needs and depending on the mathematical content they wish the children to learn.

Figure 8. Settings options for the Place Value Chart available via the settings icon.

		H	T	O
+		●●● ●●	●●● ●●●●●	●●●●● ●● ●●●●● ●
Intermediate step		○○○○○	○○○○○ ○○	○○○○○ ○○○○○ ○○○○
		○○○○○	○○○○○ ○○○○	○○○○

	H	T	O
+	3	3	8
	2	5	6
	5	8	1̲ 4
	5	9	4

	H	T	O
+	3	3	8
	2	5 1	6
	5	8	1 4
	5	9	4

Figure 7.

The following settings can be changed in the global settings of the tablet (in this case iPad).

- Show the number (with digits) above the place value chart
- Show the number word (numeral) above the place value chart
- Select the number of places: 2, 3, 4
- Select the number of places to the right of the decimal point: 0, 1, 2, 3
- Select the language
- Select the base (base 2 – base 16)
- Use the base to count and in the representation of the number with digits
- Select Montessori mode (colours of the tokens changes for each column)

3. The scope of the app—decimals, other bases, and flexible regrouping

Due to the variety of settings, it is possible to use the app in various situations and in various grade levels to learn about place value in ways that support a range of place value understandings across most of the years of primary schooling. The availability of base 2 is important in the Australian context where teachers are required to teach binary numbers as part of the Digital Technologies Curriculum.

Supporting teachers—place value chart teachers' guide

Our second contribution is supporting teachers in their teaching of Place Value. A number of researchers have indicated that a deep understanding of PV remains problematic for some teachers. Rogers (2012) suggests that gaps in students' place value knowledge may be a consequence of the routine instructional tasks provided by their teachers and a reliance on the use of standardised tests that may not accurately expose students' place value misconceptions. Hartnett (2018) found that some teachers lacked MCK and MPCK concerning PV and thus welcomed additional instruction in relation to teaching the "big ideas" underpinning PV. Hopkins and Cady (2007) indicated that some teachers have difficulties with place value including using different bases other than ten, the use of expanded notation, making conversions, and pictorially representing place value correctly. Given this lack of teacher knowledge and teacher confidence to teach PV it is important to provide further support.

In collaboration with teachers who have used the app, we have created additional support via a 25-page guide to scaffold the use of the app in their teaching.

The guide, available at http://dlgs.uni-potsdam.de/oer/stellenwerttafel-leitfaden provides detailed assistance for teachers in understanding the core PV concepts developed in the app, and how to set up the app to cater for a range of student learning opportunities. These opportunities include modifying how the columns are displayed (i.e., with or without column headings and with numerical or written headings), using the app with or without automatic bundling, extending the PV chart to include decimal fractions, and using the PV chart with bases other than 10. The guide also instructs teachers in how to use the app to teach non-standard representations of numbers, addition and subtraction, and decimal fractions. It is our view that this guide can support the technical capabilities of the app in ways that provide a digital experience for students that should assist them in developing PV understanding.

Conclusion

This article has argued that the Place Value Chart app has the potential to support the teaching of PV concepts across all primary school year levels. We have also supported teachers' MCK and MPCK by providing a 25-page teachers' guide. It is anticipated that the guide will support teachers in their teaching of PV. The adaptability of the Place Value Chart app, with its ability to incorporate decimal fractions, numbers in other bases and flexible representations, means that it has the potential to support teachers in their teaching of PV to students in all primary year levels.

Disclosure

One of the authors is the creator of the app, which is available for the lowest possible price of $0.99 from the App Store. Developer proceeds raised from app sales are used for development costs and further research and development of the app. The app is made available for free several times a year, for example usually during PME and Global Math Week.

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conference reviews

MERGA & AAMT conferences July 2019

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MERGA 42: 29 June – 4 July

The last Mathematics Education Research Group of Australia (MERGA) conference held in Western Australia was hosted in Fremantle back in 2010. It's been 25 years since MERGA was last in Perth and delegates may have noticed that in that time we've been busy upgrading the city skyline, river foreshore, as well as our conference and sporting facilities. A big thank you to everyone who made the trek across to WA to join us for the conference.

Although it has been a few years since I have been able to make it to the MERGA conference, the welcome I received was no less warm than the first conference I attended. At the time, I was struck by the fact that the MERGA delegates are such a friendly and close-knit group. While it's true that MERGA does put on a special welcome for first-time delegates, it also has a very loyal following, with some delegates not having missed a conference in 42 years!

This year's MERGA delegates heard keynote presentations from Professor Julie Sarama and Professor Doug Clements (University of Denver), Professor Jere Confrey (North Carolina State University), Professor Paul Cobb (Vanderbilt University) and Emeritus Professor Dianne Siemon. According to my sources, I believe that a record number of research papers were presented over the course of the conference. In addition, a record-breaking number of delegates attended this year's MERGA Teachers' Day.

Teachers' Day delegates heard from Associate Professor Catherine Attard (Western Sydney University, MERGA President) and Dr Paul Swan. Other Teachers' Day speakers included Emeritus Professor Doug Clarke and Anne Roche (Australian Catholic University) and Professor Barbara Clarke and Dr James Russo (Monash University), all of whom have been regular contributors to APMC.

The conference dinner was held in the River View room at Perth's new Optus Stadium. A big thank you to Lorraine Day for organising this memorable evening.